6.7 / The Fundamental Theorem of Calculus and Definite Integrals Watch the lesson ➤

F(b) - F(a) with an antiderivative; the sign is the signed area; start plus change

Learning goal

Evaluate $\int_a^b f(x)\,dx = F(b) - F(a)$ with an antiderivative, and find a value as start plus change: $f(b) = f(a) + \int_a^b f'(x)\,dx$.

Key ideas

01 The area so far is $\int_a^x f(t)\,dt = F(x) - F(a)$. Stop at $x = b$: $\int_a^b f(t)\,dt = F(b) - F(a)$. Check: $2t$ from 0 to 3: $3^2 - 0^2 = 9$, the triangle.

02 f continuous on $[a, b]$: $\int_a^b f(x)\,dx = F(b) - F(a)$, the **Fundamental Theorem** of Calculus. No limits of sums needed.

03 Read a derivative backwards: $\frac{d}{dx}x^3 = 3x^2$, so $\int_1^2 3x^2\,dx = \left[x^3\right]_1^2 = 8 - 1 = 7$. No $+C$: it drops out.

04 Start plus change: know $F(a)$ and the rate f: $F(b) = F(a) + \int_a^b f(x)\,dx$. $F(1) = 5$, $f(x) = 2x$: $F(3) = 5 + \int_1^3 2x\,dx = 5 + 8 = 13$. On the AP exam: $f(b) = f(a) + \int_a^b f'(x)\,dx$.

Remember

antiderivative
A function F with $F' = f$.

net change
$\int_a^b f'(x)\,dx = f(b) - f(a)$.

Class notes

6.7 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The start after the integral, or left out

✗ $\int_2^4 (3x^2 + 1) + 7$ (no dx); or $f(4) = 58$.

✓ $f(4) = 7 + \int_2^4 (3x^2 + 1)\,dx = 65$: start plus change.

6.7 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Evaluate with F(b) - F(a)	An antiderivative by reading a derivative backwards, then F(b) - F(a), written [F(x)] from a to b; no +C.	E01 · Practice 1, 2, 3, 4, 6
Start plus change	f(b) = f(a) + the integral from a to b of f'(x) dx.	E02, E03 · Practice 5, 7

Example 01

Video · E01

Evaluate $\int_{-2}^{2} (x^3 - 4x + 1)\,dx$. Explain the answer with the graph.

> Plan: $F(x)$, then $F(2) - F(-2)$

Example 02

Video · E02

$f(2) = 7$ and $f'(x) = 3x^2 + 1$. $f(4)$ is

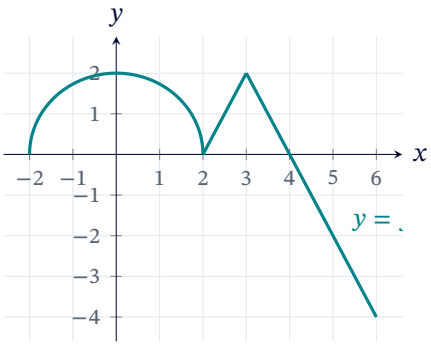
A. 58
B. 65
C. 68
D. 75

> Start plus change: the start, plus the integral of the rate.

Example 03

Video · E03

The graph of f' on $[-2, 6]$: a semicircle and two segments. $f(2) = 5$.
(a) Find $f(6)$.
(b) Find $f(-2)$.



Plan: start + change; change = area under f'

VIDEO COMPLETE Check your example answers at the end of this booklet.

6.7 / Example answers

The Fundamental Theorem of Calculus and Definite Integrals

E01 $F(x) = \dfrac{x^4}{4} - 2x^2 + x$

$F(2) - F(-2) = (4 - 8 + 2) - (4 - 8 - 2) = -2 - (-6) = 4$

Why 4: $x^3 - 4x$ is odd, its areas on $(-2, 0)$ and $(0, 2)$ cancel; 1 is even, $2\int_0^2 1\,dx = 4$.

E02 (B) $f(4) = 7 + \displaystyle\int_2^4 \left(3x^2 + 1\right) dx$

$= 7 + \left[x^3 + x\right]_2^4 = 7 + (68 - 10) = 65$

Answer B. (58 is the change alone.)

E03 $f(6) = f(2) + \displaystyle\int_2^6 f'(x)\,dx = 5 + \frac{1}{2} \cdot 2 \cdot 2 - \frac{1}{2} \cdot 2 \cdot 4 = 5 + 2 - 4 = 3$

$f(-2) = f(2) + \displaystyle\int_2^{-2} f'(x)\,dx = 5 - \int_{-2}^{2} f'(x)\,dx$

$\displaystyle\int_{-2}^{2} f'(x)\,dx = \frac{1}{2}\pi \cdot 2^2 = 2\pi \;\rightarrow\; f(-2) = 5 - 2\pi$