Lecture Notes · Unit 6 · Lecture 6 AP CED 6.6

Properties of definite integrals

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: CUT A TRIANGLE

Properties of Definite Integrals AP Calculus 6.6

walking left → sign flips

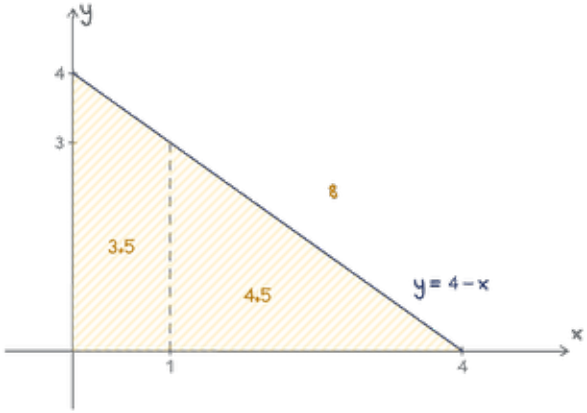
Triangle → pieces add

Cut outside?

Known integrals → new value (no f formula)

Integral = area?

SPLIT: PIECES ADD UP



$f(x) = 4 - x$ on $[0, 4]$

$\frac{1}{2}(4)(4) = 8$

Cut at $x = 1$ → $3.5 + 4.5$

$3.5 + 4.5 = 8$

$\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

b between a and c?

6.4 → $\int_a^x f(t)\,dt = F(x) - F(a)$: end minus start

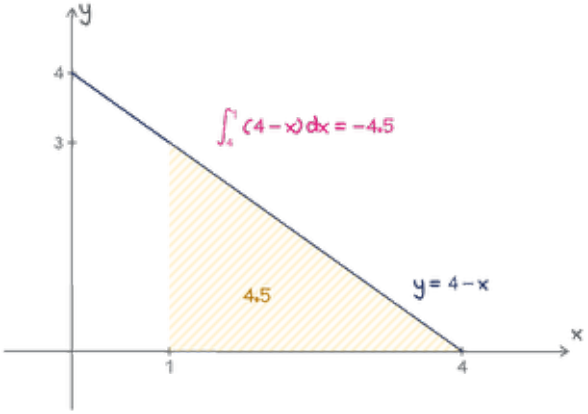
$F(b) - F(a) + F(c) - F(b)$

~~$+F(b)$~~ ~~$-F(b)$~~ → $F(c) - F(a) = \int_a^c f(x)\,dx$

b can be anywhere

f continuous on I; a, b, c in I → sufficient

FLIP THE ENDS



$4 - x \rightarrow \int_1^4 f(x)\,dx = 4.5$

$\int_4^1 f(x)\,dx = F(1) - F(4)$

$= -(F(4) - F(1)) = -4.5$

Flip → $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

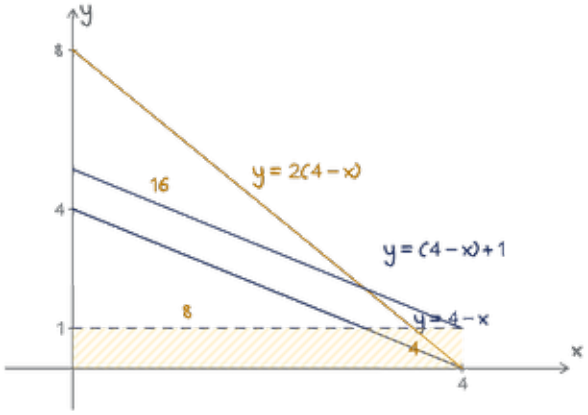
Left → −

$\int_a^a f(x)\,dx = F(a) - F(a) = 0$

width 0 → area 0

SCALE AND ADD

$\int_0^4 (4 - x)\,dx = 8$



$\int_0^4 2(4-x)\,dx = 16$

$\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

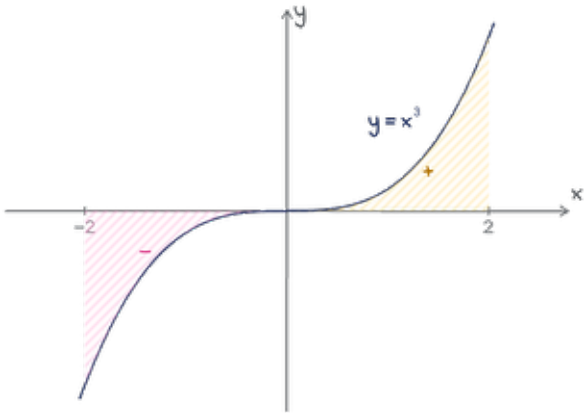
$\int_0^4 ((4-x)+1)\,dx$

$= 8 + 1 \cdot 4 = 12$

$\int_a^b (f \pm g)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g\,dx$

$\int_a^b c\,dx = c(b-a)$ → a constant is a rectangle

SYMMETRIC ENDS: ODD



x^3 $[-2,2]$

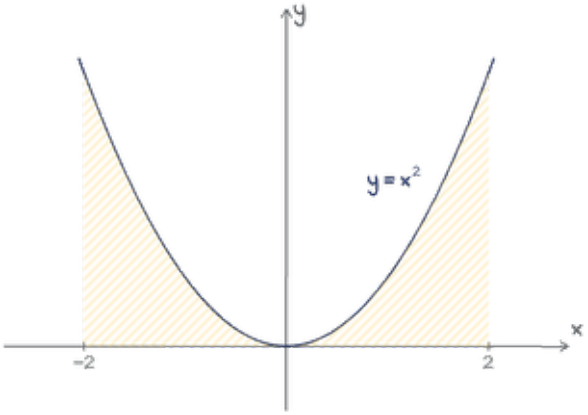
Left: below → −, right: above → +

Same size? $(-x)^3 = -x^3$

$\int_{-2}^{2} x^3\,dx = 0$

$f(-x) = -f(x)$ → odd → $\int_{-a}^{a} f(x)\,dx = 0$

SYMMETRIC ENDS: EVEN



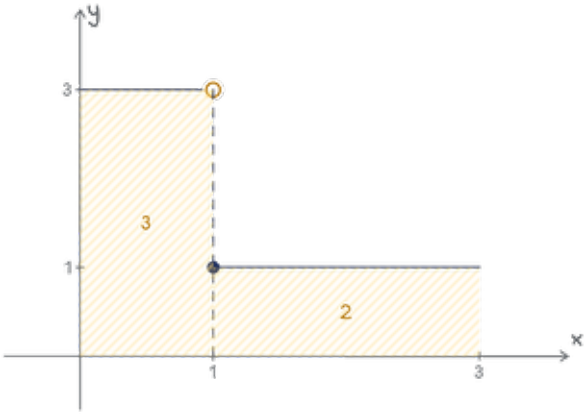
x^2 $[-2,2]$

$(-x)^2 = x^2$ → mirror image

Both above → $\int_{-2}^{2} x^2\,dx = 2\int_0^2 x^2\,dx$

$f(-x) = f(x)$ → even → $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$

A FUNCTION WITH A JUMP



Jump → $f = 3$ on $[0,1)$, $f = 1$ on $[1,3]$

$\int_0^3 f(x)\,dx = 3 \cdot 1 + 1 \cdot 2 = 5$

Jump at one x → width 0 → area 0

$\int_0^3 f(x)\,dx$ exists ✓

At $x = 1$ → f not continuous → no $F'(1) = f(1)$

ALL THE PROPERTIES ON ONE PAGE

One page: the properties

f, g continuous on 1 a, b, c in 1 interval 1, or [−a, a] → sufficient

1) $\int_a^a f(x)\,dx = 0$

2) $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$

3) $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$

4) $\int_a^b (f \pm g)\,dx = \int_a^b f\,dx \pm \int_a^b g\,dx$

5) $\int_a^c f\,dx + \int_c^b f\,dx = \int_a^b f\,dx$, c anywhere

6) f odd → $\int_{-a}^a f(x)\,dx = 0$

7) f even → $\int_{-a}^a f(x)\,dx = 2\int_0^a f(x)\,dx$

AP: a few values → find another

PROBLEM 1 OF 2

New integral values from old ones

If $\int_1^7 f(x)\,dx = 10$, $\int_4^7 f(x)\,dx = 3$, and $\int_1^4 g(x)\,dx = -2$,

find $\int_4^1 (3f(x) - 2g(x) + 1)dx$.

Solution Ends first, then split

Plan: ends first → split, scale, add

$$\int_4^1 (\cdots)\,dx = -\int_1^4 (3f(x) - 2g(x) + 1)dx$$

$$\int_1^4 f(x)\,dx = 10 - 3 = 7$$

$$3 \cdot 7 - 2 \cdot (-2) + 1 \cdot (4 - 1)$$

$$= 21 + 4 + 3 = 28$$

Answer → −28

Workbook: Problems 1, 3, 5, 6

where points vanish

X Common mistake

$$\int_4^1 (\cdots)\,dx = +28$$

$$\int_1^4 1\,dx = 1$$

✓ Do this

4 → 1 → −28

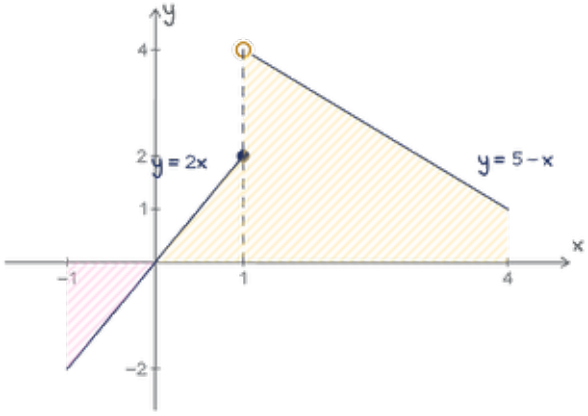
$1 \cdot (4 - 1) = 3$

PROBLEM 2 OF 2

A function with a jump

Let $f(x) = \begin{cases} 2x, & x \le 1 \\ 5 - x, & x > 1 \end{cases}$.

Find $\int_{-1}^4 f(x)\,dx$.



Solution Split at the jump

Split → signed areas

$$x = 1 \to \int_{-1}^1 2x\,dx + \int_1^4 (5 - x)\,dx$$

$$\int_{-1}^1 2x\,dx = -1 + 1 = 0$$

Not 1 + 1 = 2: below the axis counts −

$$A = \frac{1}{2}(4+1)\cdot 3 = \frac{15}{2}$$

$$0 + \frac{15}{2} = 7.5$$

$$\left|\int_{-1}^{1} 2x\,dx\right| = 0 \neq \int_{-1}^{1} |2x|\,dx = 2$$

Workbook : Problems 2, 4

WRAP-UP

Build integrals from old ones

Flip the ends → the sign flips

A constant → height × width

Condition: continuous is enough

Next → 6.7: any curve, one line

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.