6.6 / Applying Properties of Definite Integrals

Watch the lesson ➤

Split, flip, scale and add; split a function with a jump at the jump

Learning goal

Find new integral values from given ones: split, flip, scale, add a constant, and split at a jump.

Key ideas

01 $f(x) = 4 - x$ on $[0, 4]$: area 8. Split at 1: $3.5 + 4.5 = 8$. So $\int_a^b f\,dx + \int_b^c f\,dx = \int_a^c f\,dx$.

02 Flip the ends: $\int_4^1 f\,dx = F(1) - F(4) = -4.5$. So $\int_b^a f\,dx = -\int_a^b f\,dx$, and $\int_a^a f\,dx = 0$.

03 Ends $-a$ and a: split at 0. $(-x)^3 = -x^3$, so $\int_{-2}^2 x^3\,dx = 0$: $f(-x) = -f(x)$ is **odd**, $\int_{-a}^a f(x)\,dx = 0$. $(-x)^2 = x^2$, so $\int_{-2}^2 x^2\,dx = 2\int_0^2 x^2\,dx$: $f(-x) = f(x)$ is **even**, $\int_{-a}^a f = 2\int_0^a f$.

04 Heights ×2: area ×2. $\int_a^b k\,f(x)\,dx = k\int_a^b f(x)\,dx$. Add 1: $8 + 4 = 12$. $\int_a^b (f + g)\,dx = \int_a^b f\,dx + \int_a^b g\,dx$, and $\int_a^b c\,dx = c(b - a)$.

05 A jump: 3 up to 1, then 1: $\int_0^3 f\,dx = 3 \cdot 1 + 1 \cdot 2 = 5$. The jump has no width: one formula per piece.

Remember

additivity
$\int_a^b f\,dx + \int_b^c f\,dx = \int_a^c f\,dx$.

reversed limits
$\int_b^a f\,dx = -\int_a^b f\,dx$.

Class notes

6.6 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Flipped ends, a constant

✗ $\int_4^1 (\ldots)\,dx = +28$; $\int_1^4 1\,dx = 1$.

✓ Flip the ends, flip the sign: -28. $\int_1^4 1\,dx = 1 \cdot (4 - 1) = 3$.

6.6 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
New values from old ones	Flip, split, scale, add; a constant c gives c(b - a).	E01 · Practice 1, 3, 5
Areas from a graph, a jump	Shapes, below counts negative; split where the formula changes.	E02 · Practice 2, 4, 6

Example 01

Video · E01

$\int_1^7 f(x)\,dx = 10$, $\int_4^7 f(x)\,dx = 3$, $\int_1^4 g(x)\,dx = -2$.
Find $\int_4^1 (3f(x) - 2g(x) + 1)\,dx$.

> Flip the ends first; split $\int_1^7$ at 4; a constant 1 gives $1 \cdot (4 - 1)$.

Example 02

Video · E02

$$f(x) = \begin{cases} 2x, & x \le 1 \\ 5 - x, & x > 1 \end{cases}$$

Find $\int_{-1}^4 f(x)\,dx$.

> Split at the jump, $x = 1$. Use shapes on each piece.

VIDEO COMPLETE Check your example answers at the end of this booklet.

6.6 / Example answers

Applying Properties of Definite Integrals

E01 $\displaystyle\int_4^1 (\ldots)\,dx = -\int_1^4 (\ldots)\,dx$

$\displaystyle\int_1^4 f\,dx = \int_1^7 f\,dx - \int_4^7 f\,dx = 10 - 3 = 7$

$\displaystyle\int_1^4 (3f - 2g + 1)\,dx = 3(7) - 2(-2) + 1(3) = 21 + 4 + 3 = 28$

So $\int_4^1 (\ldots)\,dx = -28$.

E02 At $x = 1$: 2 from the left, 4 from the right. Split there.

$\displaystyle\int_{-1}^1 2x\,dx = -1 + 1 = 0$

$\displaystyle\int_1^4 (5 - x)\,dx = \frac{1}{2}(4 + 1) \cdot 3 = 7.5$

$\int_{-1}^4 f(x)\,dx = 0 + 7.5 = 7.5$.