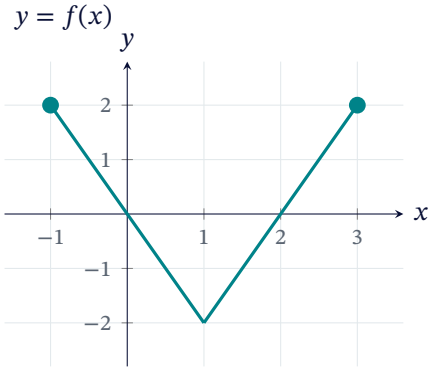
6.5 / Practice workbook

Interpreting the Behavior of Accumulation Functions

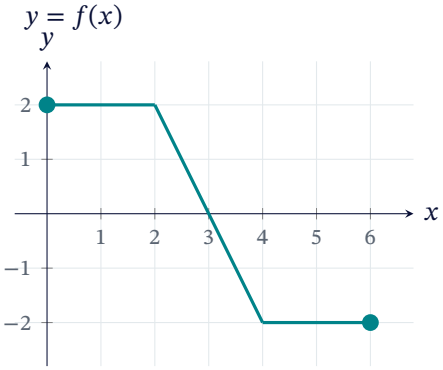
Practice

Problems 1–6 / Show your reasoning

1 The graph of f, made of two segments, is shown for $-1 \le t \le 3$. Let $g(x) = \int_{-1}^{x} f(t)\,dt$. On what intervals is g increasing? At what x does g have a relative minimum? Justify your answers.

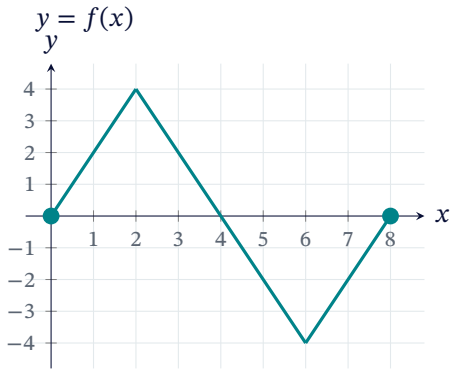


2 The graph of f is shown. Let $g(x) = \int_{0}^{x} f(t)\,dt$. Which of the following is true?



A. $g(1) < g(3) < g(6)$
B. $g(6) < g(1) < g(3)$
C. $g(6) < g(3) < g(1)$
D. $g(3) < g(1) < g(6)$

3 The graph of f, made of three segments, is shown. Let $g(x) = \int_0^x f(t)\,dt$. On which interval is the graph of g concave down?



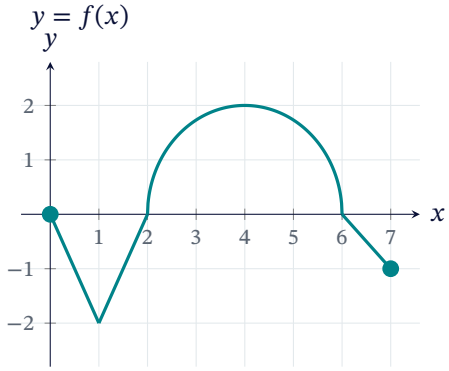
A. $(0, 2)$
B. $(2, 6)$
C. $(0, 4)$
D. $(6, 8)$

4 Let $g(x) = \int_0^x (t^2 - 4t + 3)\,dt$ for $0 \le x \le 5$.

(a) At what x does g have a relative maximum? A relative minimum? Justify your answers.

(b) Where is the graph of g concave up? At what x does g have a point of inflection? Justify your answers.

5 The graph of f consists of line segments and a semicircle of radius 2. Let $g(x) = \int_0^x f(t)\,dt$, $0 \le x \le 7$.



(a) Find $g(2)$ and $g(6)$.

(b) At what x does g have a relative minimum on $0 < x < 7$? Justify your answer.

(c) Find the absolute maximum value of g on $[0, 7]$. Justify your answer.

6 Let $g(x) = \int_1^x \ln t\,dt$ for $x > 0$. Which statements are true? I. $g(1) = 0$. II. g has a relative minimum at $x = 1$. III. The graph of g is concave down for $x > 0$.

A. I only
B. II only
C. I and II only
D. I, II, and III

6.5 / Answer key

Interpreting the Behavior of Accumulation Functions

P01 $g'(x) = f(x)$
$f > 0$ on $(-1, 0)$ and $(2, 3)$, so g is increasing there.
$g' = f$ changes from negative to positive at $x = 2$: relative minimum at $x = 2$.

P02 (B) $g(1) = 2 \cdot 1 = 2, \quad g(3) = 2 \cdot 2 + \frac{1}{2} \cdot 1 \cdot 2 = 5$
$g(6) = 5 - \frac{1}{2} \cdot 1 \cdot 2 - 2 \cdot 2 = 0$
$0 < 2 < 5$: $g(6) < g(1) < g(3)$. Answer B.

P03 (B) $g''(x) = f'(x)$
f is decreasing on $(2, 6)$, so $g'' < 0$ there: g is concave down on $(2, 6)$.
(C) is where $f > 0$: g is increasing there, not concave down. Answer B.

P04 $g'(x) = x^2 - 4x + 3 = (x - 1)(x - 3)$
g' changes from $+$ to $-$ at $x = 1$: relative max; from $-$ to $+$ at $x = 3$: relative min.
$g''(x) = 2x - 4$
$g'' > 0$ on $(2, 5)$: concave up. g'' changes sign at $x = 2$: point of inflection.

P05 $g(2) = -\frac{1}{2} \cdot 2 \cdot 2 = -2, \; g(6) = -2 + \frac{1}{2}\pi \cdot 2^2 = 2\pi - 2$
g' = f changes from negative to positive at x = 2
candidates 0, 2, 6, 7: g = 0, -2, 2pi - 2, 2pi - 5/2 -> 2pi - 2 at x = 6

P06 (C) I. $g(1) = \int_1^1 \ln t \, dt = 0$: true.
II. $g'(x) = \ln x$ changes from $-$ to $+$ at $x = 1$: true.
III. $g''(x) = \frac{1}{x} > 0$: concave up, so false. Answer C.