Lecture Notes · Unit 6 · Lecture 5 AP CED 6.5

AB 6.5 F′(x) = f(x)

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: THE GRAPH IS F, THE QUESTION IS F

The Graph is f. The Question is F. AP Calculus 6.5

where is F highest?

VALUES OF F FROM AREAS

Values of F from areas

$F(x)$ = amount piled up = $\int_0^x f(t)\,dt$

From 6.4 → $F'(x) = f(x)$

$F(1) = \frac{1}{2}(1)(2) = 1$

$F(2) = 1 + 1 = 2$

Below → $F(3) = 2 - 1 = 1$

$F(4) = 2 - 2 = 0$

$2 \to 0 \to \int_2^0 f(t)\,dt = -2$ → sign flips

F ON F: UP, DOWN, AND BEND

F on the same axes as f

Plot F → 0, 1, 2, 1, 0

F′ = f + → F ↑, − → F ↓

x = 2 → F′ = f goes + → − → rel. max

Why does F bend?

F″ = f′ → slope of f

f ↑ → F concave up, f ↓ → concave down

x = 1, 3 → inflection of F = extremum of F′ = f

Sign of f → up or down. Slope of f → bend

AREA, HEIGHT, OR SLOPE?

At x = 2 → area, height, or slope?

$F(x) = \int_0^x f(t)\,dt \to F(2), F'(2), F''(2)$?

Trap → reading all three as areas

F(2) → area → 2

F′(2) = f(2) → height → 0

F″(2) = f′(2) → slope → −2

F → area, F′ → height, F″ → slope

WHAT TO WRITE

Step 0 → F′(x) = f(x)

F peaks at 2 as F′ = f goes + → −.

At x = 1 F′ = f rises then falls.

f has a relative max at x = 1

Name f in every reason → not 'it', not 'the graph'

Exam letter g → g′(x) = f(x)

Rel. max ≠ absolute max → compare candidates

PROBLEM 1 OF 2

An accumulation function from a graph

The graph of f has segments and a semicircle.

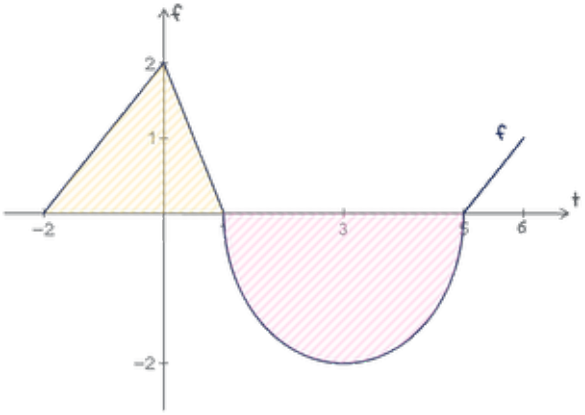
Let $g(x) = \int_{-2}^{x} f(t)\,dt$.

(a) Find $g(1)$ and $g(5)$.

(b) Where is g increasing? Where does g have a relative maximum? Justify.

(c) Find the inflection points of g. Justify.

(d) Find the absolute minimum of g on $[-2, 6]$. Justify.



Solution (a) Values from areas

(a)

Plan: $g' = f$ → behavior, areas → values

$g(1)$: $[-2, 1]$ → area above

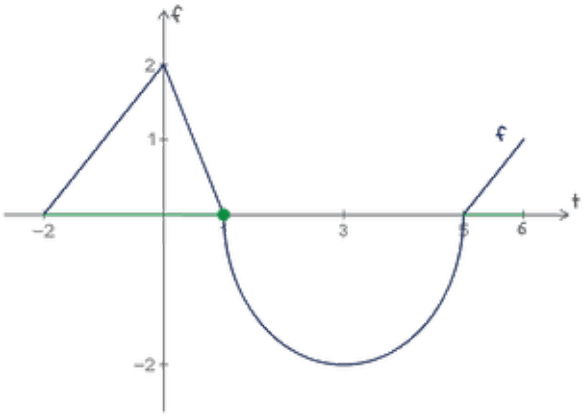
$g(1) = \frac{1}{2}(3)(2) = 3$

$5 - 1 = 4$ → radius 2 → area

$\frac{1}{2}\pi(2)^2 = 2\pi$

$g(5) = 3 - 2\pi$

Solution (b) Increasing and relative max



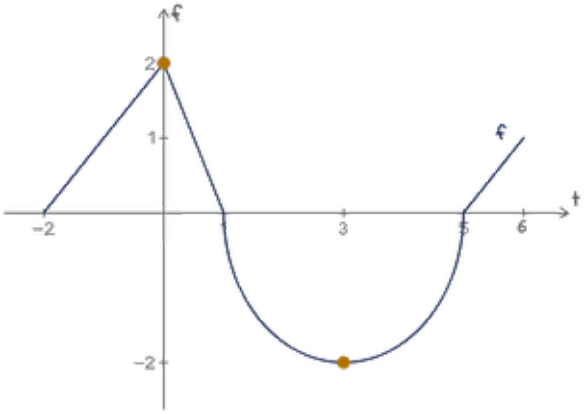
(b)

Step 0 → $g'(x) = f(x)$

$f > 0$ on $(-2, 1), (5, 6)$ → g increasing

$x = 1$ → g' = f changes + to − → rel. max

Solution (c) Inflection points



(c)

Turns of f → peak $x = 0$, bottom $x = 3$

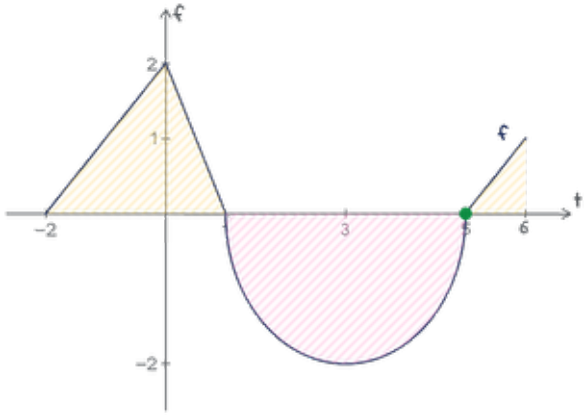
$x = 0$ → g' = f rises then falls

$x = 3$ → g' = f falls then rises

g' peaks at 0, dips at 3

g inflections → 0, 3

Solution (d) Absolute minimum: compare candidates



(d)

Candidates −2, 1, 5, 6 → ends or $g' = f = 0$

$g(-2) = 0, \quad g(1) = 3$

$g(5) = 3 - 2\pi$

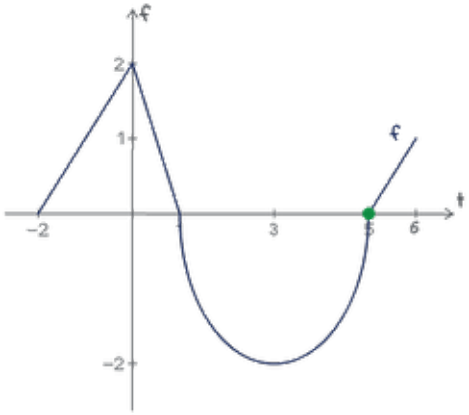
$g(6) = 3 - 2\pi + \frac{1}{2} = 3.5 - 2\pi$

$2\pi > 6 \rightarrow 3 - 2\pi < -3$, the smallest

Abs. min $3 - 2\pi$ at $x = 5$

Workbook → graph of f, behavior of g

g′(5) → area or height?



X Common mistake

-2π (area)

f ↓ → g ↓

✓ Do this

$g'(5) = f(5) = 0$

$(1,5) \rightarrow g' = f < 0 \rightarrow g \downarrow$

PROBLEM 2 OF 2

A relative minimum, no integrating

Let $g(x) = \int_1^x \frac{t^2 - 2t - 3}{t^2 + 1}\,dt$. At what value of x does g have a relative minimum?

(A) −1 (B) 1 (C) 3 (D) none

Solution Read the sign of g'

Plan: no integral → g′(x) = integrand → sign

$$g'(x) = \frac{x^2 - 2x - 3}{x^2 + 1}$$

$$= \frac{(x-3)(x+1)}{x^2 + 1}$$

$x^2 + 1 > 0$ always → the top decides

$x < -1 \rightarrow +, \quad -1 < x < 3 \rightarrow -, \quad x > 3 \rightarrow +$

$x = 3 \rightarrow g'$ changes − to + → rel. min

$x = -1 \rightarrow +$ to − → rel. max, not asked

Solution Read the sign of g'

So (C)

Workbook → sign of g′

WRAP-UP

Where is F highest?

This graph → highest F(2) = 2

Sign → up or down; slope → bend

First line → F′ = f or g′ = f

fs name in every reason

walking left flips the sign

Next → 6.6 → flip, split, scale

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.