6.5 / Interpreting the Behavior of Accumulation Functions

Watch the lesson ➤

F' = f: where F rises, peaks and bends, read from the graph of f; values of F from areas

Learning goal

Read an accumulation function $F(x) = \int_a^x f(t)\,dt$ from the graph of f (values from areas, where F rises, peaks and bends), with $F'(x) = f(x)$ written first.

Key ideas

01 Only the graph of f: $F(x) = \int_0^x f(t)\,dt$ is the signed area so far. f up to 2 at 1, down to -2 at 3: $F(1) = 1, F(2) = 2, F(3) = 1, F(4) = 0$. $f > 0$: F rises; $f < 0$: F falls.

02 $F' = f$: at $x = 2$, f changes from + to −, so F peaks there.

03 $F'' = f'$, the slope of f. f rising: F concave up; f falling: concave down. Where f turns, F has **inflection points**.

04 What to write: $F'(x) = f(x)$ first (its own point). Max: "$F' = f$ changes from + to − at $x = 2$." Inflection: "$F' = f$ changes from increasing to decreasing at $x = 1$." Name f, not "the graph" or "it".

Remember

point of inflection of g

Where $F'' = f'$ changes sign: where the graph of f turns from rising to falling or back.

Class notes

6.5 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 An area for a slope, or 'the graph'

✗ $F'(5)$ = the area from 1 to $5 = -2\pi$; or "the graph goes down, so F decreases.'

✓ $F'(5) = f(5) = 0$, a height. "$F' = f < 0$ on $(1, 5)$, so F is decreasing.'

6.5 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

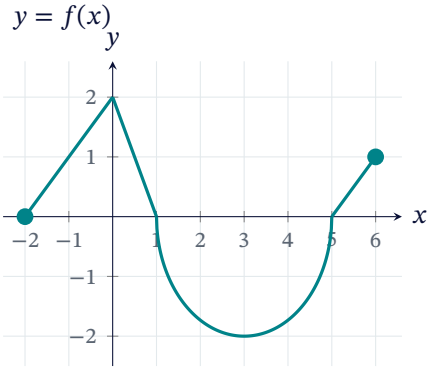
Read the problem, name the type, then use its method

Type	Method	Problems
Rises, falls, peaks	g' = f: f > 0 -> g increasing; f changes sign -> relative extremum. Name f in the reason.	E02 · Practice 1
Bends	g" = f': f increasing -> g concave up; f turns -> inflection point of g.	Practice 3, 4, 6
Values and the largest or smallest	Values of g are signed areas; compare the candidates (ends and where f changes sign).	E01 · Practice 2, 5

Example 01

Video · E01

Graph of f: segments, a semicircle. $g(x) = \int_{-2}^{x} f(t)\,dt$
(a) g(1), g(5)
(b) g increasing where? Relative max? Justify.
(c) Inflection points of g? Justify.
(d) Absolute minimum of g on $[-2, 6]$? Justify.



Write $g'(x) = f(x)$ first. Values of g are areas under f.

Example 02

Video · E02

$g(x) = \int_1^x \frac{t^2-2t-3}{t^2+1}\,dt$. g has a relative minimum at $x =$

A. -1
B. 1
C. 3
D. none

g' is the integrand at x. The bottom is always positive: look at the top.

VIDEO COMPLETE Check your example answers at the end of this booklet.

6.5 / Example answers

Interpreting the Behavior of Accumulation Functions

E01 $g(1) = \frac{1}{2} \cdot 2 \cdot 2 + \frac{1}{2} \cdot 1 \cdot 2 = 3, \quad g(5) = 3 - \frac{1}{2}\pi \cdot 2^2 = 3 - 2\pi$

(b) $g'(x) = f(x) > 0$ on $(-2, 1)$ and $(5, 6)$: g is increasing there. $g' = f$ changes from $+$ to $-$ at $x = 1$: relative maximum.

(c) $g' = f$ changes from increasing to decreasing at $x = 0$ and from decreasing to increasing at $x = 3$: points of inflection.

(d) Candidates $-2,\ 1,\ 5,\ 6$: $g = 0,\ 3,\ 3 - 2\pi,\ 3.5 - 2\pi$. The smallest is $3 - 2\pi$, at $x = 5$.

E02 (C) $g'(x) = \dfrac{x^2 - 2x - 3}{x^2 + 1} = \dfrac{(x - 3)(x + 1)}{x^2 + 1}$

The bottom is positive, so the sign is the top's: $+,\ -,\ +$. It changes from $-$ to $+$ at $x = 3$.

Answer C.