6.4 / Practice workbook

The Fundamental Theorem of Calculus and Accumulation Functions

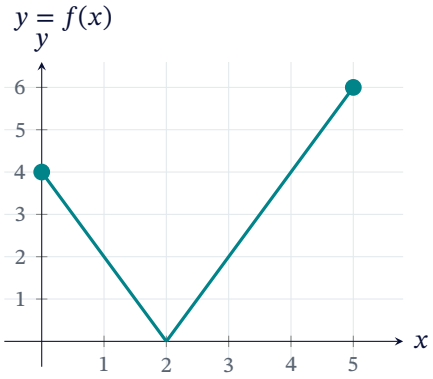
Practice

Problems 1–6 / Show your reasoning

1 $\dfrac{d}{dx}\int_2^{3x}\sqrt{1+t^3}\,dt =$

A. $\sqrt{1+27x^3}$
B. $3\sqrt{1+27x^3}$
C. $\sqrt{1+x^3}$
D. $3\sqrt{1+x^3}$

2 The graph of f on $0 \le t \le 5$, made of two segments, is shown. Let $g(x) = \int_0^x f(t)\,dt$. Find $g(4)$ and $g'(4)$, and write an equation of the line tangent to the graph of g at $x = 4$.



3 The function f is continuous, with values in the table. If $G(x) = \int_0^{x^2} f(t)\,dt$, what is $G'(3)$?

t	1	3	9
$f(t)$	4	5	2

A. 2
B. 6
C. 12
D. 30

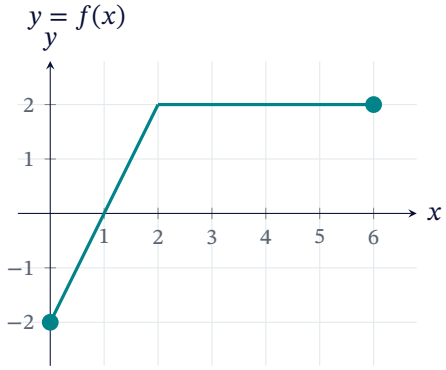
4 Let $H(x) = \int_1^{e^x} \ln t \, dt$.

(a) Find $H(0)$.

(b) Find $H'(x)$ in simplest form.

(c) Find $H''(0)$.

5 The graph of f is shown for $0 \le t \le 6$. Let $g(x) = \int_0^{2x} f(t) \, dt$ for $0 \le x \le 3$.



(a) Find $g(2)$.

(b) Find $g'(2)$.

(c) Find the value of x, $0 < x < 3$, where $g'(x) = 0$.

6 Let f be continuous. If $\int_0^{x^2} f(t)\,dt = x^4 + 2x^2$ for all x, then $f(4) =$

A. 6
B. 10
C. 20
D. 40

6.4 / Answer key

The Fundamental Theorem of Calculus and Accumulation Functions

P01 (B) $f(t) = \sqrt{1+t^3}$, upper end $3x$.
$\frac{d}{dx}\int_2^{3x} f(t)\,dt = f(3x)\cdot 3 = 3\sqrt{1+27x^3}$
Answer B.

P02 $g(4) = \frac{1}{2}\cdot 2\cdot 4 + \frac{1}{2}\cdot 2\cdot 4 = 8$
$g'(4) = f(4) = 4$
Tangent line: $y = 8 + 4(x-4)$.

P03 (C) $G'(x) = f(x^2)\cdot 2x$
$G'(3) = f(9)\cdot 6 = 2\cdot 6 = 12$
Answer C.

P04 $H(0) = \int_1^1 \ln t\,dt = 0$
$H'(x) = \ln(e^x)\cdot e^x = xe^x$
$H''(x) = e^x + xe^x \;\rightarrow\; H''(0) = 1$

P05 $g(2) = \int_0^4 f(t)\,dt = -1 + 1 + 4 = 4$
$g'(x) = f(2x)\cdot 2 \;\rightarrow\; g'(2) = 2f(4) = 4$
$f(2x) = 0 \;\rightarrow\; 2x = 1 \;\rightarrow\; x = \frac{1}{2}$

P06 (B) $f(x^2)\cdot 2x = 4x^3 + 4x$
$f(x^2) = 2x^2 + 2$
$x^2 = 4:\; f(4) = 8 + 2 = 10$
Answer B.