6.3 / Practice workbook

Riemann Sums, Summation Notation, and Definite Integral Notation

Practice

Problems 1–6 / Show your reasoning

1 Write $\int_0^{\pi} \sin x \, dx$ as the limit of a right Riemann sum with n subintervals of equal width.

2 $\lim_{n\to\infty} \sum_{k=1}^{n} e^{\frac{k}{n}} \cdot \frac{1}{n}$ is equal to

A. $\int_0^1 e^x \, dx$

B. $\int_1^2 e^x \, dx$

C. $\int_0^n e^x \, dx$

D. $n \int_0^1 e^x \, dx$

3 What is the value of $\lim_{n\to\infty} \sum_{k=1}^{n} \left(1 + \frac{3k}{n}\right) \frac{3}{n}$?

A. $\frac{9}{2}$

B. $\frac{15}{2}$

C. 8

D. 15

4 Let $f(x) = 9 - x^2$ on $[0, 3]$.

(a) Find the right Riemann sum with three subintervals of equal width.

(b) Write $\int_0^3 (9 - x^2)\,dx$ as the limit of a right Riemann sum with n equal subintervals.

5 Which of the following is NOT equal to $\lim_{n\to\infty} \sum_{k=1}^{n} \ln\left(1 + \frac{3k}{n}\right)\frac{3}{n}$?

A. $\int_1^4 \ln x\,dx$

B. $\int_0^3 \ln(1 + x)\,dx$

C. $\int_1^4 \ln(1 + x)\,dx$

D. the limit of the right Riemann sums of $\ln x$ on $[1, 4]$

6 Which of the following limits is equal to $\int_2^6 \frac{1}{x}\,dx$?

A. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2+\frac{4k}{n}} \cdot \frac{2}{n}$

B. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{\frac{4k}{n}} \cdot \frac{4}{n}$

C. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{4}{2n+4k}$

D. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{2+\frac{k}{n}} \cdot \frac{4}{n}$

6.3 / Answer key

Riemann Sums, Summation Notation, and Definite Integral Notation

P01 $\Delta x = \dfrac{\pi - 0}{n} = \dfrac{\pi}{n}, \quad x_k = \dfrac{\pi k}{n}$

$$\int_0^{\pi} \sin x \, dx = \lim_{n\to\infty} \sum_{k=1}^{n} \sin\left(\frac{\pi k}{n}\right)\frac{\pi}{n}$$

P02 (A) $\Delta x = \dfrac{1}{n} \;\to\; b - a = 1, \qquad x_k = \dfrac{k}{n}$: 0 to 1

x_k goes into e^x: $\int_0^1 e^x \, dx$. Answer A.

P03 (B) $\Delta x = \dfrac{3}{n}, \quad x_k = 1 + \dfrac{3k}{n}$: 1 to 4, $\quad f(x) = x$

$$\int_1^4 x \, dx = \tfrac{1}{2}(1+4)\cdot 3 = \tfrac{15}{2}$$

Answer B.

P04 width 1, right ends 1, 2, 3 : $1\cdot 8 + 1\cdot 5 + 1\cdot 0 = 13$

$\Delta x = \dfrac{3}{n}, \quad x_k = \dfrac{3k}{n}$

$$\int_0^3 (9 - x^2)\, dx = \lim_{n\to\infty} \sum_{k=1}^{n} \left(9 - \left(\frac{3k}{n}\right)^2\right)\frac{3}{n}$$

P05 (C) Read $x_k = 1 + \frac{3k}{n}$: $\int_1^4 \ln x \, dx$, so (A) and (D) are equal to it.

Read $x_k = \frac{3k}{n}$: $\int_0^3 \ln(1+x)\, dx$, so (B) is equal too.

(C) moves the start to 1 and adds 1 again: not equal. Answer C.

P06 (C) $\Delta x = \dfrac{6-2}{n} = \dfrac{4}{n}, \quad x_k = 2 + \dfrac{4k}{n}$

$$\frac{1}{2 + \frac{4k}{n}} \cdot \frac{4}{n} = \frac{4}{2n + 4k}$$

Answer C.