6.2 / Practice workbook

Approximating Areas with Riemann Sums

Practice

Problems 1–6 / Show your reasoning

1 Oil is pumped into a tank at a rate of $P(t)$ gallons per hour. Selected values are in the table. Use a left Riemann sum with the four subintervals of the table to estimate the gallons pumped in from $t = 0$ to $t = 7$.

t (hours)	0	1	3	4	7
$P(t)$ (gal/h)	6	9	10	8	5

2 Let $f(x) = x^2 + 1$. What is the right Riemann sum for f on $[0, 3]$ with three subintervals of equal width?

A. 8
B. 11.75
C. 12.5
D. 17

3 People enter a park at a rate of $V(t)$ people per hour, $0 \le t \le 12$. Selected values are in the table.

t (hours)	0	2	4	6	8	10	12
$V(t)$ (people/h)	40	60	90	120	100	70	30

(a) Estimate the number of people who enter, using a midpoint sum with three equal subintervals.

(b) Estimate the same number with a trapezoidal sum on the same subintervals.

4 For which function is the left Riemann sum on $[0, 2]$, with any number of subintervals, an overestimate of the integral?

A. x^2
B. e^{-x}
C. $\sin\left(\frac{\pi x}{2}\right)$
D. $\ln(x+1)$

5 Water is pumped out of a tank at a rate of $W(t)$ liters per minute, where W is increasing. Selected values are in the table.

t (min)	0	3	5	9	12
$W(t)$ (L/min)	4	7	8	13	15

(a) Estimate $\int_0^{12} W(t)\,dt$ with a right Riemann sum.

(b) Is this estimate an overestimate or an underestimate? Explain.

(c) Find two numbers that $\int_0^{12} W(t)\,dt$ must lie between.

6 Selected values of f are in the table. The trapezoidal sum for $\int_0^7 f(x)\,dx$ with the three subintervals of the table is 32. What is k?

x	0	2	3	7
$f(x)$	3	k	6	4

A. 4
B. 6
C. 8
D. 12

6.2 / Answer key

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P01 Widths 1, 2, 1, 3; left heights 6, 9, 10, 8.
$1 \cdot 6 + 2 \cdot 9 + 1 \cdot 10 + 3 \cdot 8 = 6 + 18 + 10 + 24 = 58$
About 58 gallons.

P02 (D) width $= \dfrac{3-0}{3} = 1$, right ends $x = 1,\ 2,\ 3$
$1 \cdot 2 + 1 \cdot 5 + 1 \cdot 10 = 17$
Answer D.

P03 $4 \cdot 60 + 4 \cdot 120 + 4 \cdot 70 = 1000$
$\frac{1}{2}(40 + 90) \cdot 4 + \frac{1}{2}(90 + 100) \cdot 4 + \frac{1}{2}(100 + 30) \cdot 4 = 900$

P04 (B) f decreasing $\Rightarrow$ the left end is the highest on every piece $\Rightarrow$ left sum over.
e^{-x} is decreasing. x^2 and $\ln(x+1)$ increase (under); $\sin \frac{\pi x}{2}$ goes up, then down (cannot tell). Answer B.

P05 $3 \cdot 7 + 2 \cdot 8 + 4 \cdot 13 + 3 \cdot 15 = 134$
W increasing -> the right end is the highest on each piece -> overestimate
$3 \cdot 4 + 2 \cdot 7 + 4 \cdot 8 + 3 \cdot 13 = 97 \ \rightarrow\ 97 < \displaystyle\int_0^{12} W(t)\,dt < 134$

P06 (A) $\frac{1}{2}(3 + k) \cdot 2 + \frac{1}{2}(k + 6) \cdot 1 + \frac{1}{2}(6 + 4) \cdot 4 = 32$
$(3 + k) + \frac{k+6}{2} + 20 = 32 \ \rightarrow\ 3k = 12 \ \rightarrow\ k = 4$
Answer A.