Lecture Notes · Unit 6 · Lecture 2 AP CED 6.2

Riemann sums from a table

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INTRO: SNOW ON A CLIPBOARD

Riemann Sums from a Table AP Calculus 6.2

Curved rate, 4 numbers

Snow readings at t = 0, 2, 3, 6 h

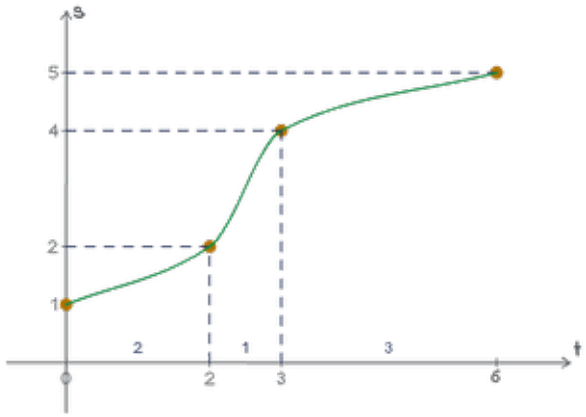
Gaps: 2, 1, 3 h → uneven

How much snow?

4 ways → over or under? Why?

IDEA 1: LEFT AND RIGHT RIEMANN SUMS

Snow from a table



Rate s(t), inches per hour

t	0	2	3	6
s(t)	1	2	4	5

Widths: 2 − 0 = 2, 3 − 2 = 1, 6 − 3 = 3

Between → unknown → any climb

$\frac{in}{h} \times h = in$ → area under s

Not exact → cut at table times

Left: $2 \cdot 1 + 1 \cdot 2 + 3 \cdot 4$

$= 2 + 2 + 12 = 16$ s(6) = 5 unused

Right: $2 \cdot 2 + 1 \cdot 4 + 3 \cdot 5$

$= 4 + 4 + 15 = 23$ s(0) = 1 unused

2, 1, 3 → OK?

1 term = 1 rectangle, own width

Not $\frac{6-0}{3} = 2 \rightarrow 2(1+2+4) = 14 \neq 16$ ✗

Any widths → left, right Riemann sums

Join points → trapezoids

$\frac{\text{left} + \text{right}}{2} \times \text{width}$

$\frac{1+2}{2} \cdot 2 + \frac{2+4}{2} \cdot 1 + \frac{4+5}{2} \cdot 3$

$= 3 + 3 + 13.5 = 19.5$ → trapezoidal sum

Check: $\frac{16+23}{2} = 19.5$ (same pieces)

Over or under? Table can't promise

Given: s increasing → left end lowest

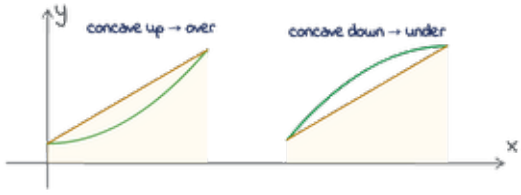
→ 16 underestimate

Right end highest → 23 over

Below axis: −5 < −2 → still under

Decreasing → flips up, down → can't tell

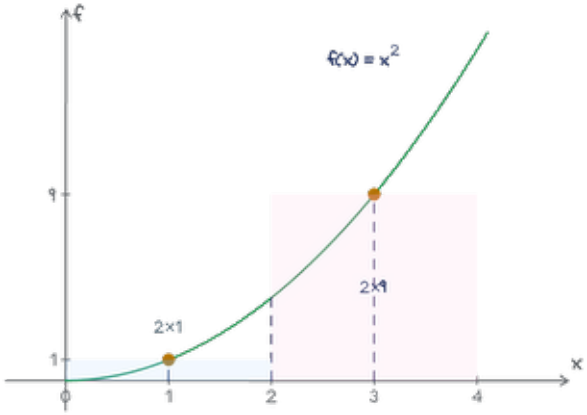
Trapezoids → the bend



s increasing so left sum is under.

IDEA 4: MIDPOINT RIEMANN SUM

With a formula: midpoints



$f(x) = x^2$ on $[0, 4]$, 2 pieces → width 2

Middles: $x = 1, x = 3$

$2 \cdot 1^2 + 2 \cdot 3^2$

$= 2 + 18 = 20$ → midpoint Riemann sum

PROBLEM 1 OF 3

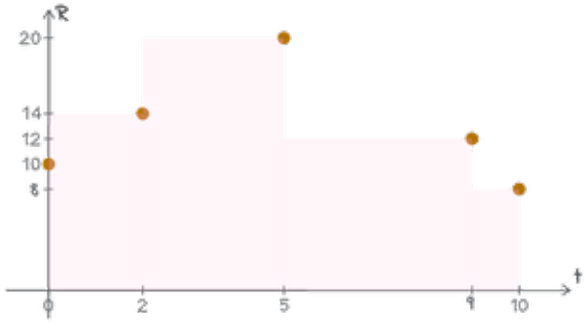
Cars into a garage, from a table

Cars enter a garage at a rate of $R(t)$ cars per hour, $0 \le t \le 10$.

(a) Estimate the number of cars that enter with a right Riemann sum, using the 4 subintervals in the table.

(b) Estimate the same number with a trapezoidal sum, same subintervals.

t	0	2	5	9	10
$R(t)$	10	14	20	12	8



Solution (a) Right ends × widths

(a)

Plan: widths → width × right height

Widths 2, 3, 4, 1

$2 \cdot 14 + 3 \cdot 20 + 4 \cdot 12 + 1 \cdot 8$

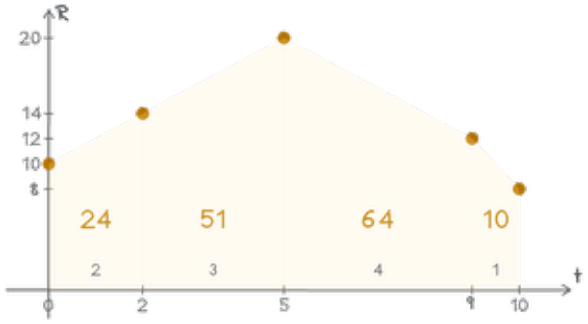
$R(0)$ → unused

$28 + 60 + 48 + 8 = 144$

cars/h × h = cars → ≈ 144 (garage: start + in − out)

Solution (b) Average of the two heights × width

t	0	2	5	9	10
$R(t)$	10	14	20	12	8



(b)

Plan: $\frac{\text{left} + \text{right}}{2} \times \text{width}$

$\frac{10+14}{2} \cdot 2 + \frac{14+20}{2} \cdot 3 +$

$\frac{20+12}{2} \cdot 4 + \frac{12+8}{2} \cdot 1$

$= 149$

≈ 149 cars

Workbook → uneven widths

Points go missing

✗ Common mistake

≈ 144 only → AP: 2 points lost

$\frac{10}{4} = 2.5$ for every width

✓ Do this

$2 \cdot 14 + \cdots \to 144$

widths → 2, 3, 4, 1

Calculator → sum first

PROBLEM 2 OF 3

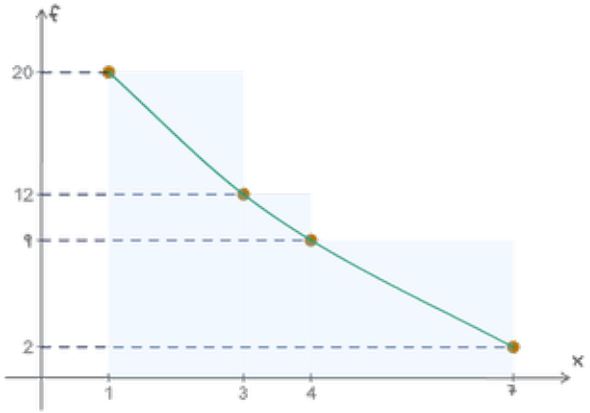
A decreasing f: left sum, over or under

f is decreasing on $[1, 7]$.

$f(1) = 20,\ f(3) = 12,\ f(4) = 9,\ f(7) = 2.$

(a) Use a left Riemann sum with 3 subintervals to approximate $\int_1^7 f(x)\,dx$.

(b) Is it an overestimate or underestimate? Explain.



Solution (a) Left ends × widths

(a)

Plan: width × left height

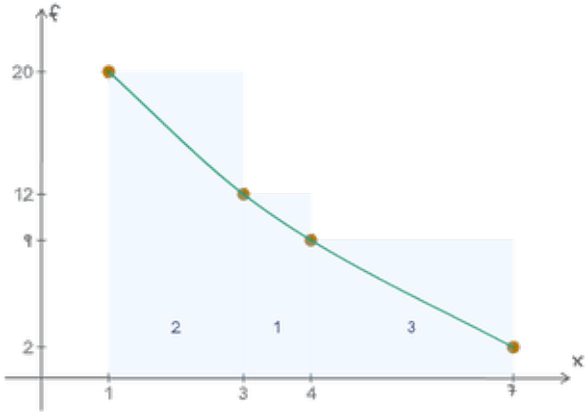
widths 2, 1, 3

$2 \cdot 20 + 1 \cdot 12 + 3 \cdot 9$

$40 + 12 + 27 = 79$ f(7) = 2 unused

$\int_1^7 f(x)\,dx \approx 79$

Solution (b) Decreasing → over



(b)

Plan: use the given fact

f decreasing (given) → left end highest

f decreasing so left sum 79 is an over estimate.

Workbook → reason sentence

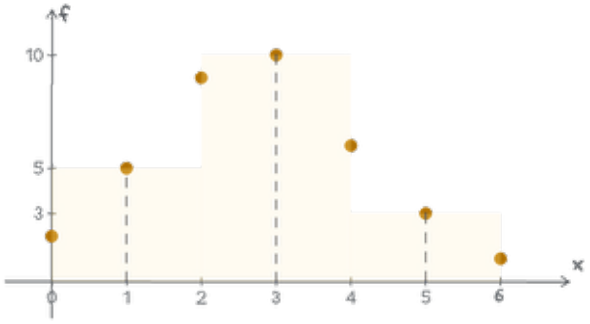
PROBLEM 3 OF 3

A midpoint sum from a table

The table gives values of f. The midpoint Riemann sum for f on $[0, 6]$ with three subintervals of equal width is

(A) 18 (B) 32 (C) 34 (D) 36

x	0	1	2	3	4	5	6
f(x)	2	5	9	10	6	3	1



Solution Width, the middles in the table, then add

Plan: width, mids, heights

$$\text{width } \frac{6-0}{3} = 2$$

$[0,2]$, $[2,4]$, $[4,6]$ → mid 1, 3, 5

Heights → 5, 10, 3 (not 0, 2, 4, 6)

$$2 \cdot 5 + 2 \cdot 10 + 2 \cdot 3 = 36$$

So (D) 36

Workbook → width, middles first

WRAP-UP

How much snow?

widths first, from the table

width × height, each piece → show the sum

over or under → only with a given reason

Next → 6.3: thinner and thinner pieces?

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.