6.2 / Approximating Areas with Riemann Sums

Watch the lesson ➤

Left, right, midpoint and trapezoidal sums from a table or a formula; over or under

Learning goal

Estimate an integral with left, right, midpoint and trapezoidal sums from a table or a formula, show the sum, and tell over or under from increasing or decreasing.

Key ideas

01 Snow at $s(t)$ in/h, known only at $t = 0,\ 2,\ 3,\ 6$ ($s = 1,\ 2,\ 4,\ 5$). Left heights: $2 \cdot 1 + 1 \cdot 2 + 3 \cdot 4 = 16$. Right heights: $2 \cdot 2 + 1 \cdot 4 + 3 \cdot 5 = 23$. The widths can differ. These are **Riemann sums**.

02 A trapezoid on each piece: $\frac{1}{2}(1+2) \cdot 2 + \frac{1}{2}(2+4) \cdot 1 + \frac{1}{2}(4+5) \cdot 3 = 19.5 = \frac{16+23}{2}$, the average of the two rectangles: the **trapezoidal sum**.

03 With a formula you can use the middle: $g(x) = x^2$ on $[0, 4]$, middles 1 and 3: $2 \cdot 1^2 + 2 \cdot 3^2 = 20$, the **midpoint sum**.

04 f increasing: the left end is the lowest, so the left sum **under**estimates and the right sum overestimates. Decreasing: the other way. Up and down: it cannot tell.

05 Show the sum (width × height for each piece), then the number.

Remember

Riemann sum
Width times a height of f on each piece, added. Left, right or midpoint: which x gives the height.

trapezoidal sum
The sum of $\frac{1}{2}$(left + right) · width over the pieces; the average of the left and right sums.

Class notes

6.2 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The number only, or one width for all

✗ ≈ 144, with no sum shown; or the same width 2.5 for every piece.

✓ Write width × height for each piece, with the widths from the table (2, 3, 4, 1), then the number.

6.2 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
A Riemann sum from a table	Width times the chosen height for each piece, then add. Show the sum, then the number.	E01 · Practice 1, 3, 6
A Riemann sum from a formula	Width = (b - a)/n; the left, right or middle x of each piece; add width times f.	E03 · Practice 2
Over or under	Increasing: left under, right over. Decreasing: the other way. Give the reason that was given.	E02 · Practice 4, 5

Example 01

Video · E01

Cars enter a garage at a rate of $R(t)$ cars per hour, $0 \le t \le 10$.
(a) Estimate the number of cars that enter with a right Riemann sum, using the 4 subintervals in the table.
(b) Estimate the same number with a trapezoidal sum, same subintervals.

t (hours)	0	2	5	9	10
$R(t)$ (cars/h)	10	14	20	12	8

> The widths come from the table: 2, 3, 4, 1. Width times height for each piece, then add.

Example 02

Video · E02

f is decreasing on $[1, 7]$. $f(1) = 20$, $f(3) = 12$, $f(4) = 9$, $f(7) = 2$.
(a) Left Riemann sum, 3 subintervals, to approximate $\int_1^7 f(x)\,dx$
(b) Overestimate or underestimate? Explain.

> Decreasing: which end of each piece is the highest?

Example 03

Video · E03

The table gives values of f. The midpoint Riemann sum for f on $[0, 6]$ with three subintervals of equal width is

x	0	1	2	3	4	5	6
$f(x)$	2	5	9	10	6	3	1

A. 18
B. 32
C. 34
D. 36

Plan: width, middles in the table, add

VIDEO COMPLETE Check your example answers at the end of this booklet.

6.2 / Example answers

Approximating Areas with Riemann Sums

E01 $2 \cdot 14 + 3 \cdot 20 + 4 \cdot 12 + 1 \cdot 8 = 28 + 60 + 48 + 8 = 144$

(a) About 144 cars.

$\frac{1}{2}(10 + 14) \cdot 2 + \frac{1}{2}(14 + 20) \cdot 3 + \frac{1}{2}(20 + 12) \cdot 4 + \frac{1}{2}(12 + 8) \cdot 1 = 24 + 51 + 64 + 10 = 149$

(b) About 149 cars.

E02 Widths $2,\ 1,\ 3$.

$2 \cdot 20 + 1 \cdot 12 + 3 \cdot 9 = 40 + 12 + 27 = 79$

f is decreasing, so the left end is the highest on each piece: the rectangles are above the curve, and 79 is an overestimate.

E03 (D) width $= \dfrac{6 - 0}{3} = 2$

Pieces $[0, 2]$, $[2, 4]$, $[4, 6]$: middles $x = 1, 3, 5$, heights $f(1) = 5$, $f(3) = 10$, $f(5) = 3$.

$2 \cdot 5 + 2 \cdot 10 + 2 \cdot 3 = 36$

So (D) 36.