Lecture Notes · Unit 6 · Lecture 1 AP CED 6.1

Accumulation of change

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: FROM HOW FAST TO HOW MUCH

Accumulation of Change AP Calculus 6.1

Until now → how fast?

Today: flip it

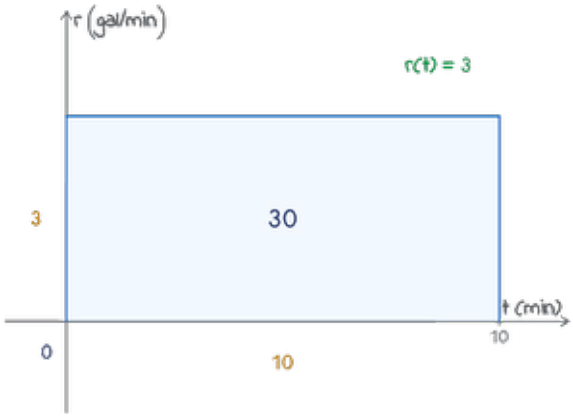
3 gal/min × 10 min → 30 gal

Changing rate → how much?

Rate graph → amount + sentence

IDEA 1: A FLAT RATE

A flat rate = a rectangle

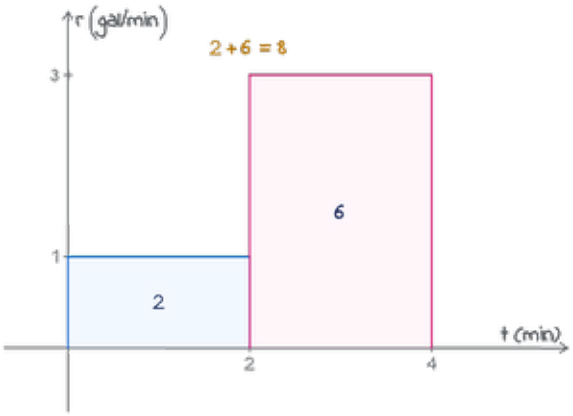


Flat at 3

3 × 10 = rectangle → 30 gal

$\frac{\text{gal}}{\text{min}} \times \text{min} = \text{gal}$ → amount

Height 3 = a rate → gallons = area



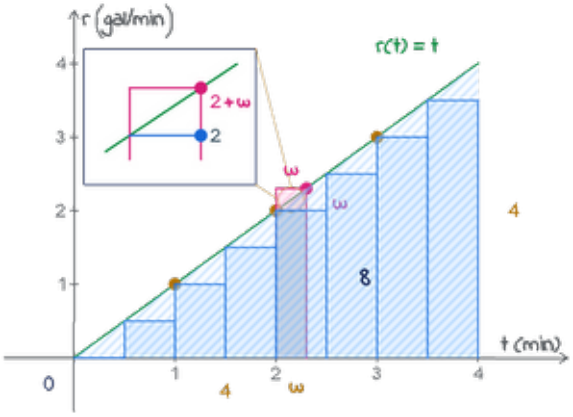
Rate 1 for 2 min, then 3 for 2 min

1 × 2 = 2, 3 × 2 = 6 → 8 gal

Amount = area under the rate graph

IDEA 2: A RATE THAT NEVER SITS STILL

A changing rate → thin strips



r(t) = t → r(1) = 1, r(3) = 3

No one rate → rate × time fails

ω from t = 2 → rate 2 to 2 + ω

Low 2ω, high (2 + ω)ω → water between

Gap: ω high, ω wide

→ $\omega \cdot \omega = \omega^2$

Tiny $w \to w^2 \approx 0 \to$ water $\approx$ rate $\times w$

Strips, 0 to 4 → nearly fill

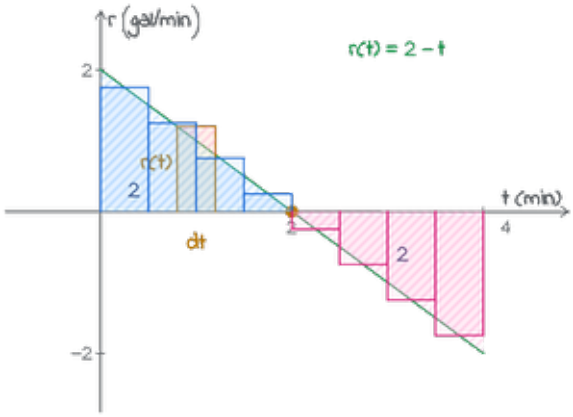
Area $= \frac{1}{2} \cdot 4 \cdot 4 = 8$ gal

Thinner → closer

Area = accumulated change

IDEA 3: BELOW THE AXIS

Below the axis → net change



$r(t) = 2 - t \to 0$ at $t = 2 \to r < 0$

In $\frac{1}{2} \cdot 2 \cdot 2 = 2$, out, same size → 2

Areas positive → $2 + 2 = 4$?

Drained → below: −

Net change $= 2 - 2 = 0$ gal

Total $2 + 2 = 4$ gal

$v \to 0$ displacement, 4 total distance

Start 10 → end $10 + 0 = 10$ gal

Strip: height $r(t)$, tiny width dt

Corner sticks out → tiny → $\approx$

$r(t)\,dt \approx$ water in that strip

Long S = sum → $\int_0^4 r(t)\,dt$ from 0 to 4

→ definite integral = net change

net change in the water,

$t = 0$ to $t = 4$ min → 0 gallons

Area → gal, not gal/min

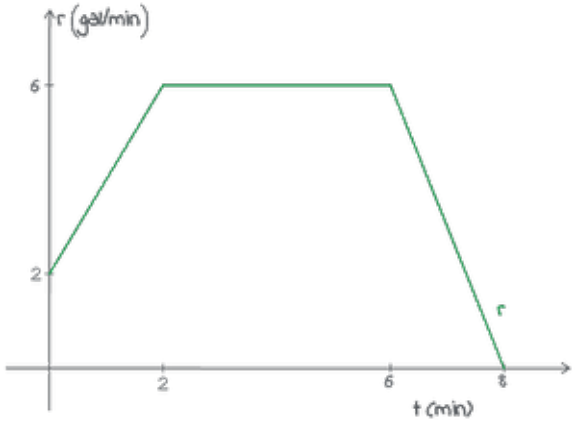
PROBLEM 1 OF 2

From a rate graph to an amount

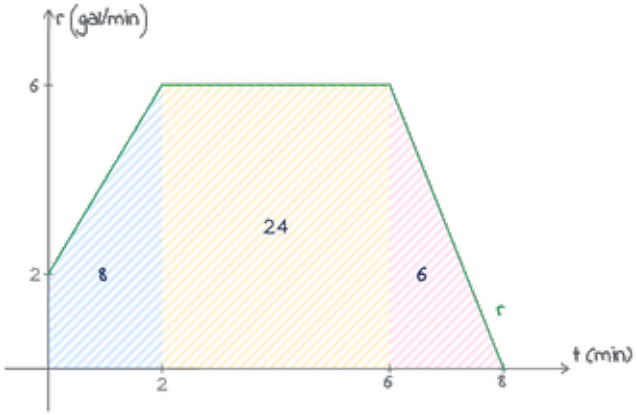
Water flows into a tank at $r(t)$ gallons per minute. The graph of r is shown, $0 \le t \le 8$.

(a) From $t = 0$ to $t = 8$, how many gallons flow in?

(b) Find $\int_2^8 r(t)\,dt$. Explain its meaning, with units.



Solution (a) Water = area, by shapes



(a)

Plan: water = area under r → friendly shapes

$0 \to 2$ trapezoid $\frac{1}{2}(2 + 6) \cdot 2 = 8$

$2 \to 6$ rectangle $6 \cdot 4 = 24$

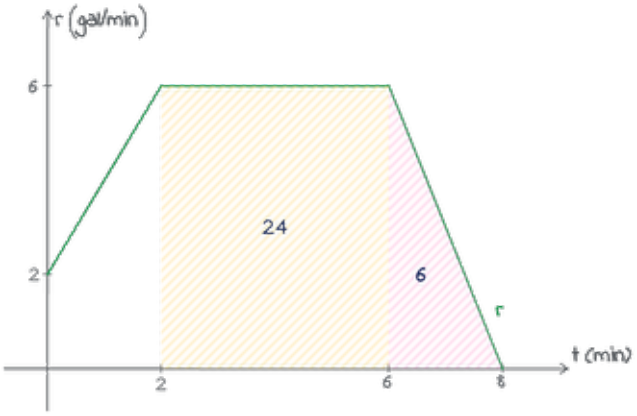
$6 \to 8$ triangle $\frac{1}{2} \cdot 2 \cdot 6 = 6$

$8 + 24 + 6 = 38$

So 38 gallons.

(b): you first → integral, sentence

Solution (b) The integral, then its sentence



(b)

Plan: area 2 to 8 → sentence

$$\int_2^8 r(t)\,dt = 24 + 6 = 30$$

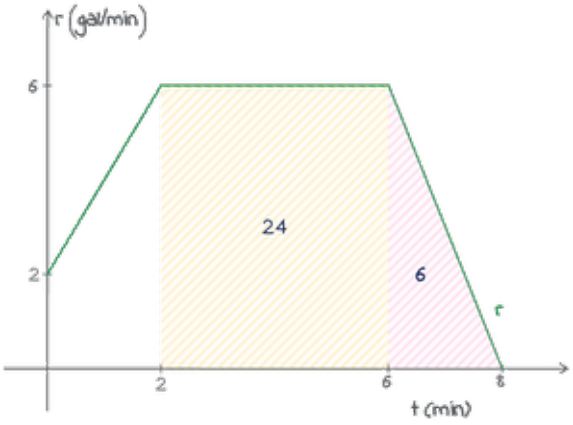
30 gal flowed in,

$t = 2$ to $t = 8$ min.

Workbook → your own graph → the sentence

COMMON MISTAKE: AN AMOUNT READ AS A RATE

Careful → points go missing



$\int_2^8 r(t)\,dt = 30$ gal/min

Area multiplies → gal

→ 30 gallons

30 gallons flowed in. When?

→ $t = 2$ to $t = 8$ min

AP 2023 Q1: amount and interval

PROBLEM 2 OF 2

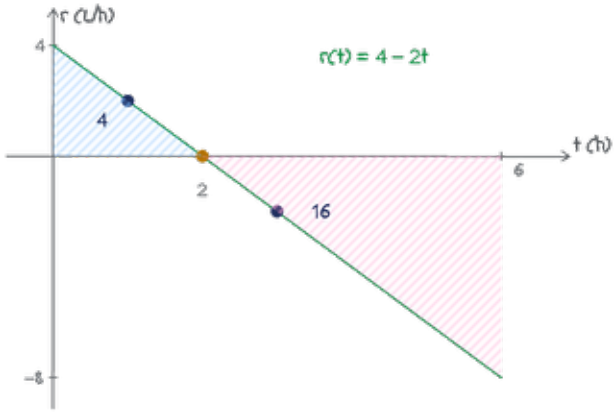
A pond that fills, then drains

Water flows into a pond at $r(t) = 4 - 2t$ liters per hour, $0 \le t \le 6$. A negative rate means water leaves. At $t = 0$ the pond holds 50 liters. At $t = 6$, how many liters are in the pond?

(A) 30 (B) 38 (C) 54 (D) 70

t	0	1	3	6
$r(t)$	4	2	−2	−8

Solution Draw the rate; above adds, below subtracts



Plan: draw r; above adds, below subtracts

$4 - 2t = 0 \rightarrow t = 2$

$0 \rightarrow 2$ in $\frac{1}{2} \cdot 2 \cdot 4 = 4$

$2 \rightarrow 6$ out $\frac{1}{2} \cdot 4 \cdot 8 = 16$

change $= 4 - 16 = -12$

Not done → start 50

$50 + (-12) = 38$

So (B) → 38 liters.

(D) $70 = 50 + 4 + 16$; (C) 54 → omits 16

Workbook → draw r first

WRAP-UP

Changing rate → how much?

Net change = above − below

End = start + change

$\int_a^b r(t)\,dt$ → amount + interval, not a rate

Next → 6.2 Riemann sums

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.