6.1 / Exploring Accumulations of Change

Watch the lesson ➤

The amount is the area under the rate graph; below the axis counts negative; the meaning sentence

Learning goal

Read an amount from a rate graph as the area under it, count area below the axis as a decrease, and say what $\int_a^b r(t)\,dt$ means, with units and the interval.

Key ideas

01 Constant rate: rate × time. 3 gal/min for 10 min is 30 gal, and $\frac{\text{gal}}{\text{min}} \times \text{min} = \text{gal}$, an amount. On the rate graph that is a rectangle: the amount is the **area** under the rate graph.

02 A changing rate, $r(t) = t$ from $t = 2$: in a short time w the water is between $2w$ and $(2 + w)w$; the gap is w^2, which shrinks fast. Thin strips fill the region, so the water is the area: $\frac{1}{2} \cdot 4 \cdot 4 = 8$ gal on $[0, 4]$. This is the **accumulated change**.

03 Below the axis the rate is negative: water comes out. $r(t) = 2 - t$ on $[0, 4]$: in 2, out 2, net change 0; starting at 10 gal it ends at 10 gal.

04 Each strip is $r(t)\,dt$; adding them from 0 to 4 gives the **definite integral** $\int_0^4 r(t)\,dt$, the net change in the amount.

05 Its meaning: an amount (rate unit × time unit) and the interval: "net change in the water, from $t = 0$ to $t = 4$ min, 0 gallons."

Remember

accumulated change
The total change in an amount over an interval: the area under its rate graph (below the axis counts negative).

definite integral
$\int_a^b r(t)\,dt$: the sum of $r(t)\,dt$ from $t = a$ to $t = b$, the signed area under r.

Class notes

6.1 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Read as a rate

✗ $\int_2^8 r(t)\,dt = 30$ gal/min.

✓ The area of a rate is an amount: 30 gallons flowed in, from $t = 2$ to $t = 8$ minutes. Say the interval.

6.1 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

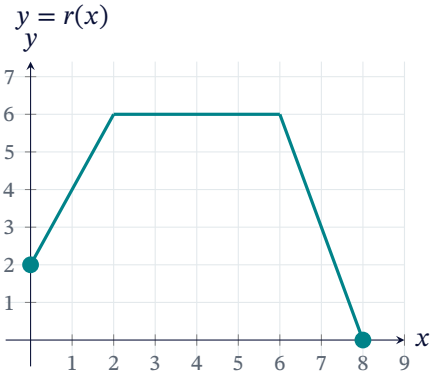
Read the problem, name the type, then use its method

Type	Method	Problems
Amount from a rate graph	The amount is the area under the rate graph: cut it into rectangles, triangles, trapezoids.	E01 · Practice 1, 3
Above adds, below takes away	Find where the rate is zero; the area above the axis adds, the area below subtracts. Start plus net change.	E02 · Practice 5, 6
The meaning sentence, with units	An amount (units: rate unit times time unit) and the time interval.	Practice 2, 4

Example 01

Video · E01

Water flows into a tank at $r(t)$ gallons per minute. The graph of r is shown, $0 \le t \le 8$.
(a) From $t = 0$ to $t = 8$, how many gallons flow in?
(b) Find $\int_2^8 r(t)\,dt$. Explain its meaning, with units.



The water is the area under r: cut it into a trapezoid, a rectangle and a triangle.

Example 02

Video · E02

Water flows into a pond at $r(t) = 4 - 2t$ liters per hour, $0 \le t \le 6$. A negative rate means water leaves. At $t = 0$ the pond holds 50 liters. At $t = 6$, how many liters are in the pond?

A. 30
B. 38
C. 54
D. 70

> Draw r. Where is it zero? Above the axis adds, below subtracts.

VIDEO COMPLETE Check your example answers at the end of this booklet.

6.1 / Example answers

Exploring Accumulations of Change

E01 $0 \to 2:\ \frac{1}{2}(2+6)\cdot 2 = 8, \quad 2 \to 6:\ 6\cdot 4 = 24, \quad 6 \to 8:\ \frac{1}{2}\cdot 2\cdot 6 = 6$

(a) $8 + 24 + 6 = 38$ gallons flow in.

$$\int_2^8 r(t)\,dt = 24 + 6 = 30$$

(b) 30 gallons of water flowed into the tank from $t = 2$ to $t = 8$ minutes.

E02 (B) $r(t) = 4 - 2t = 0 \ \to\ t = 2$

$0 \to 2:$ in $\frac{1}{2}\cdot 2\cdot 4 = 4, \quad 2 \to 6:$ out $\frac{1}{2}\cdot 4\cdot 8 = 16$

$50 + 4 - 16 = 38$

Answer B.