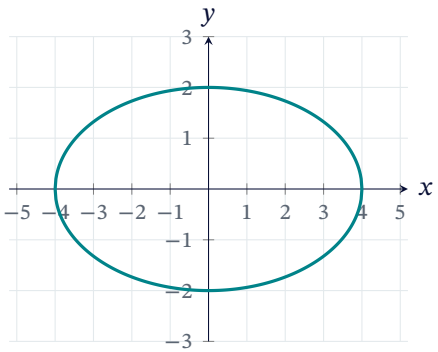
5.12 / Practice workbook

Exploring Behaviors of Implicit Relations

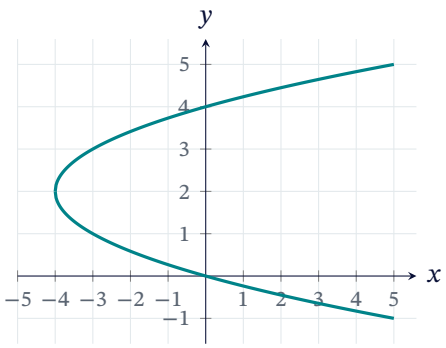
Practice

Problems 1–7 / Show your reasoning

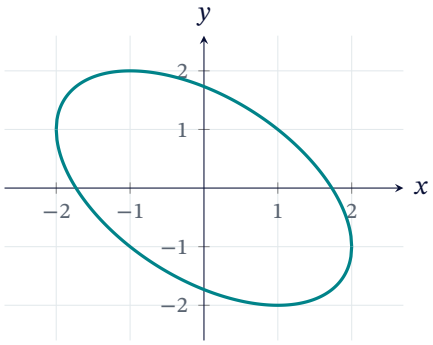
1 Find all points on the ellipse $x^2 + 4y^2 = 16$ where the tangent line is horizontal, and where it is vertical.



2 Find the point on the curve $x = y^2 - 4y$ where the tangent line is vertical.



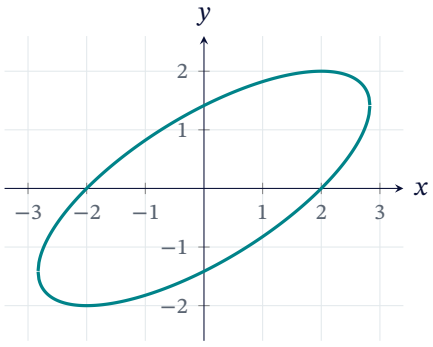
3 For the curve $x^2 + xy + y^2 = 3$, $\frac{dy}{dx} = -\frac{2x + y}{x + 2y}$. At which point on the curve is the tangent line horizontal?



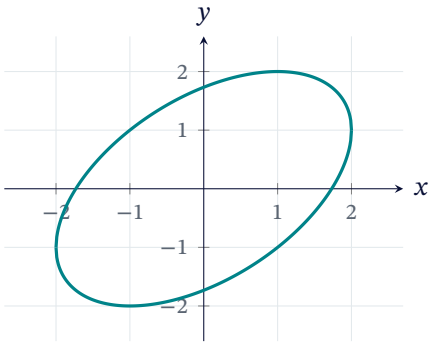
A. $(1, -2)$
B. $(1, 1)$
C. $(2, -1)$
D. $(-2, 1)$

4 For the circle $x^2 + y^2 = 25$, $\frac{dy}{dx} = -\frac{x}{y}$. Find $\frac{d^2y}{dx^2}$ in terms of y. Where is the circle concave down?

5 For the curve $x^2 - 2xy + 2y^2 = 4$, find the points where $\frac{dy}{dx} = 0$. Use $\frac{d^2y}{dx^2}$ to decide whether each is a relative maximum or a relative minimum.



6 Find all points on the curve $x^2 - xy + y^2 = 3$ where the tangent line is horizontal, and where it is vertical.



5.12 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

The curve $y^3 - 3y = x^2$ from Example 1 has $\frac{dy}{dx} = \frac{2x}{3y^2 - 3}$. Find every point on the curve where the tangent line is vertical. Show that each point is on the curve.

5.12 / Answer key

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1 horizontal at $(0, \pm 2)$; vertical at $(\pm 4, 0)$

2 $(-4, 2)$

3 (A)

4 $\dfrac{d^2y}{dx^2} = -\dfrac{25}{y^3}$; concave down for $y > 0$

5 $(2, 2)$ relative max; $(-2, -2)$ relative min

6 horizontal at $(1, 2)$, $(-1, -2)$; vertical at $(2, 1)$, $(-2, -1)$

7 bottom $3y^2 - 3 = 0$: $y = \pm 1$; $y = 1$ gives $x^2 = -2$ (no point); $y = -1$ gives $x = \pm\sqrt{2}$, top $2x \neq 0$