5.11 / Practice workbook

Solving Optimization Problems

Practice

Problems 1–7 / Show your reasoning

1 Find the minimum value of $f(x) = 2x + \frac{8}{x}$ for $x > 0$. Justify your answer.

2 The numbers $x \geq 0$ and $y \geq 0$ satisfy $x + 2y = 12$. Find the largest possible value of xy. Justify your answer.

3 The function f is differentiable on $(0, \infty)$, and $f'(3) = 0$. Which statement justifies that f has an absolute minimum at $x = 3$ on $(0, \infty)$?

A. $f'(3) = 0$

B. f' changes from negative to positive at $x = 3$

C. $x = 3$ is the only critical point on $(0, \infty)$, and f' changes from negative to positive there

D. f is continuous at $x = 3$

4 An open box with a square base must hold 32 in^3. Find the dimensions that use the least material. Justify your answer.

5 A rectangular garden has an area of 200 ft^2. The fence on the front side costs 6 dollars per foot, and the fence on the other three sides costs 2 dollars per foot. Find the dimensions that make the cost as small as possible. Justify your answer.

6 A person in a boat is 3 km from the nearest point P on a straight shore. The person wants to reach a point Q on the shore 4 km from P, rowing at 3 km/h and walking at 5 km/h. The person lands x km from P, with $0 \leq x \leq 4$. Find the least possible travel time. Justify your answer.

5.11 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

A rumor spreads through a school. For $0 \le t \le 10$ days, the number of students who have heard it is $N(t) = 60t^2 - 4t^3$.

(a) At what time t is the number of students who have heard the rumor growing fastest? Justify your answer.

(b) How fast is the number growing at that time? Indicate units of measure.

5.11 / Answer key

Solving Optimization Problems

1 8, at $x = 2$

2 18, at $x = 6, y = 3$

3 (C)

4 base 4 in by 4 in, height 2 in

5 10 ft (front) by 20 ft, cost 160 dollars

6 $\frac{8}{5}$ h, landing $\frac{9}{4}$ km from P

7 (a) $t = 5$: maximize $N'(t) = 120t - 12t^2$ on $[0, 10]$; $N'(0) = 0, N'(5) = 300, N'(10) = 0$; (b) $N'(5) = 300$ students per day