Lecture Notes · Unit 5 · Lecture 11 AP CED 5.11

Solving Optimization Problems

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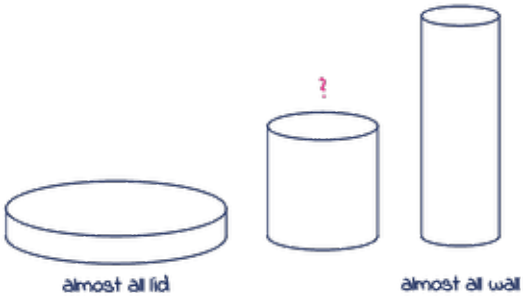
INTRO: THE CAN WITH NO ENDS

Prove the Optimum AP Calculus 5.11



$V = 16\pi$ → pancake: all lid

Straw → wall · between → least metal

$r > 0$ → no ends

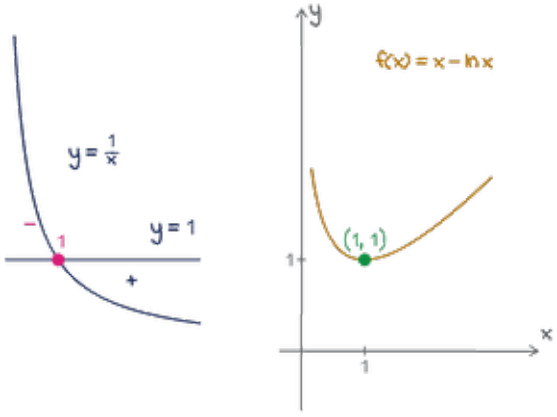
$r = 0$? ✗ $r = \infty$? ✗

Closed → compare (box → 0, 128, 0)

No ends → one sentence

NO ENDS: ONE CRITICAL POINT

No ends: $f(x) = x - \ln x$



$f(x) = x - \ln x$, $x > 0$

$f'(x) = 1 - \frac{1}{x}$

$y = 1 - y = \frac{1}{x}$

Meet once → $x = 1$

Left of 1 → $f' < 0$, right → $f' > 0$

f falls for $x < 1$, rises for $x > 1$

One turn only → abs. min $f(1) = 1$

PICK THE ARGUMENT

Closed domain · compare $f' = 0$, f' undefined, and endpoints

No ends · 1 critical point

→ the only one + sign change (or f'')

Several → check f' throughout

f continuous on $x > 0$,

$x = 1$ is the only critical point,

f' − → + there → abs. min at $x = 1$

ANSWER WHAT WAS ASKED

Answer → the quantity asked, with units

One peak or valley → compare nearby counts ≥ 0

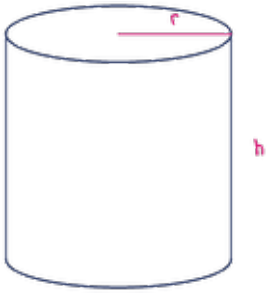
Greatest · least · fastest

Least → min. Fastest → max rate.

PROBLEM 1 OF 2

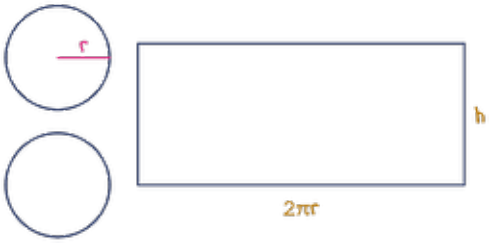
The can

A closed cylindrical can holds 16π cubic inches. Find the radius and height with the least surface area. Justify that it is a minimum.



Solution Set it up: open the can

$V = \pi r^2 h$ $S = 2\pi r^2 + 2\pi rh$



Plan: S in r alone, domain → $S' = 0$, then the sign

top + bottom → $2\pi r^2$

side → $2\pi r \cdot h$

$\pi r^2 h = 16\pi \rightarrow h = \frac{16}{r^2}$

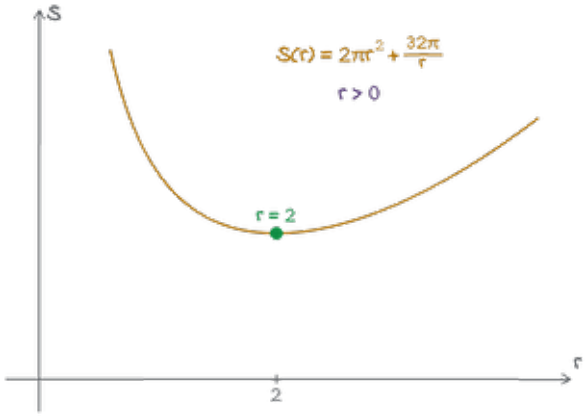
$S(r) = 2\pi r^2 + 2\pi r \cdot \frac{16}{r^2}$

$\frac{32\pi r}{r} \rightarrow S = 2\pi r^2 + \frac{32\pi}{r}$

$r > 0$ → open, no ends

Your turn → $S' = 0$

Solution Find the critical point



$S'(r) = 4\pi r - \frac{32\pi}{r^2}$

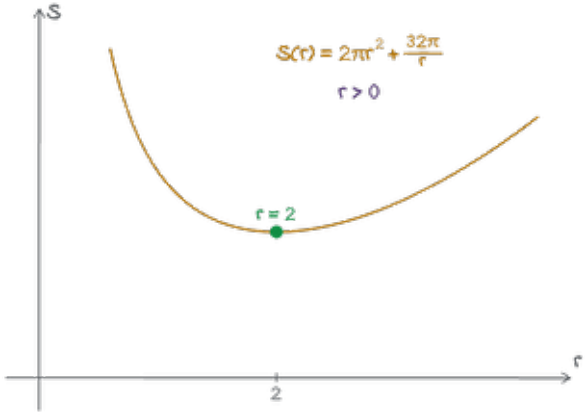
$= \frac{4\pi(r^3 - 8)}{r^2}$

$= 0 \rightarrow r^3 = 8 \rightarrow r = 2$, the only one

$r^2 > 0$, $4\pi > 0$ → sign of $r^3 - 8$

− before 2 · + after

Solution Prove it, then answer



S continuous on $r > 0$,

$r = 2$ is the only critical point,

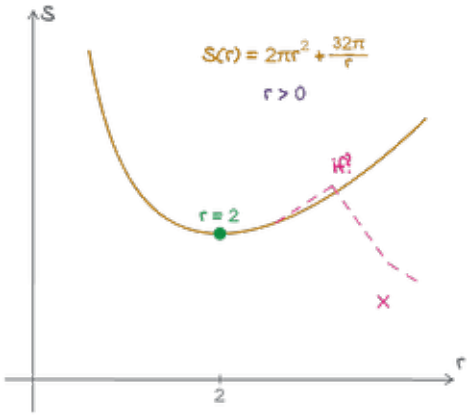
S' − → + there → abs. min at $r = 2$

Asked → r and h → $h = \frac{16}{2^2} = 4$

$r = 2$ in, $h = 4$ in

Workbook open box: 32 in³

Sign change = absolute min?



X Common mistake

$S'(2) = 0 \rightarrow$ min?

$- \rightarrow + \rightarrow$ local

Another turn?

✓ Do this

No second critical point

Only one $+ - \rightarrow +$

PROBLEM 2 OF 2

Growing fastest

A new song is released. For $0 \le t \le 3$ days, the total number of downloads is $N(t) = t^3 - 3t^2 + 4t$ thousand.

At what time are the downloads growing fastest? How fast are they growing then? Justify your answer.

Solution Which quantity?

No 'optimize' → which quantity?

Try $N' = 0 \rightarrow 3t^2 - 6t + 4 = 0$?

$= 3(t-1)^2 + 1 > 0 \rightarrow$ never 0

Fastest → greatest rate

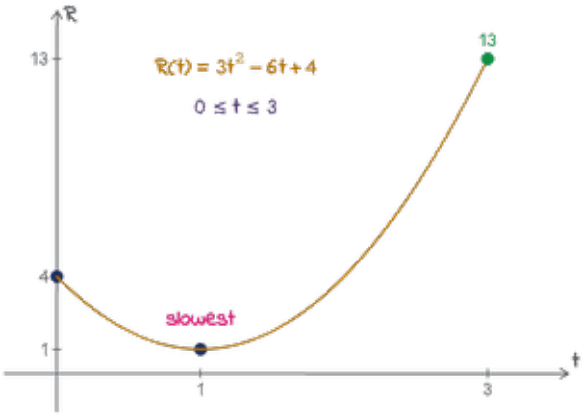
how fast they grow

$= R(t) = N'(t) = 3t^2 - 6t + 4$

$0 \le t \le 3 \rightarrow$ closed → compare

Your turn: $R' = 0 \rightarrow$ compare R

Solution Compare the rates



$R'(t) = 6t - 6 = 0 \rightarrow t = 1$

$R(0) = 4,\ R(1) = 1,\ R(3) = 13$

$t = 1 \rightarrow$ slowest; right end → highest

$t = 3$ days → 13 thousand per day

Workbook closed domain

WRAP-UP

Prove it, then answer

Closed → compare; no ends → only

Answer the quantity asked

No 'optimize' → find the quantity

5.12 → beyond $y = f(x)$