5.11 / Solving Optimization Problems

Watch the lesson ➤

Proving the biggest or the smallest

Learning goal

Justify that an optimization answer is the largest or smallest: compare candidates on a closed domain, or use the only critical point on an open one.

Key ideas

01 Closed domain: compare. $P = x(10 - x)$ on $[0, 10]$: $P' = 0$ at 5; candidates $0,\ 5,\ 10$ give $0,\ 25,\ 0$, so the max is 25 (the Candidates Test).

02 Open domain, one critical point: $f(x) = x - \ln x$, $x > 0$: $f'(x) = 1 - \frac{1}{x}$, only $x = 1$, and it is a relative min, so it is the **absolute min**, $f(1) = 1$.

03 The sentence: "$x = 1$ is the only critical point, and f' changes from $-$ to $+$ there, so f has an absolute minimum at $x = 1$."

04 More than one critical point on an open domain: look at f' on the whole domain. Then answer what was asked, with units.

Remember

only critical point

On an interval, a single critical point that is a relative extremum is also absolute.

Class notes

5.11 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 A sign change is only relative

✗ $S'(2) = 0$ and S' changes sign, so S has an absolute minimum at $r = 2$.

✓ $r = 2$ is the **only** critical point on $r > 0$, and S' changes from $-$ to $+$ there.

5.11 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Open domain, the only critical point	One critical point, its sign change, then the sentence with ”only”.	E01 · Practice 1, 3, 4, 5
Closed domain, compare	The critical points inside and both ends; the largest or the smallest value.	E02 · Practice 2, 6, 7

Example 01

Video · E01

A closed cylindrical can holds 16π cubic inches.
Find the radius and height with the least surface area. Justify that it is a minimum.

> $r > 0$ is open: no ends to compare. Say “the only critical point” and give its sign change.

Example 02

Video · E02

A new song is released. For $0 \le t \le 3$ days, the total number of downloads is $N(t) = t^3 - 3t^2 + 4t$ thousand.
At what time are the downloads growing fastest? How fast? Justify.

> Plan: which quantity? → closed → compare

VIDEO COMPLETE Check your example answers at the end of this booklet.

5.11 / Example answers

Solving Optimization Problems

E01 $r = 2in, h = 4in$

E02 (a) 3; (b) 13