5.10 / Practice workbook

Introduction to Optimization Problems

Practice

Problems 1–6 / Show your reasoning

1 Two nonnegative numbers have a sum of 12. Find the numbers that make the product of the square of one number and the other number as large as possible.

2 A farmer has 240 m of fence for a rectangular pen. A fence parallel to one side divides the pen into two parts. What dimensions give the largest total area?

3 An open box is made from a 10 in by 16 in sheet by cutting a square of side x from each corner and folding up the sides. What is the domain of the volume function $V(x)$?

A. $x > 0$
B. $0 \le x \le 5$
C. $0 \le x \le 8$
D. $0 \le x \le 10$

4 Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(2, 0)$.

5 The open box from the 10 in by 16 in sheet has volume $V(x) = x(10 - 2x)(16 - 2x)$ for $0 \le x \le 5$. Find the largest possible volume. Justify your answer.

6 A rectangle has its base on the x-axis and its upper corners on the semicircle $y = \sqrt{4 - x^2}$. Find the largest possible area of the rectangle.

5.10 / Answer key

Introduction to Optimization Problems

P01 $P(x) = x^2(12 - x), \qquad 0 \le x \le 12$
$P'(x) = 24x - 3x^2 = 3x(8 - x) = 0 \;\rightarrow\; x = 0,\ 8$
$P(0) = 0, \quad P(8) = 256, \quad P(12) = 0$
The numbers are 8 and 4 (product 256).

P02 $3x + 2y = 240 \;\rightarrow\; y = 120 - \frac{3}{2}x, \qquad A(x) = x\left(120 - \frac{3}{2}x\right), \quad 0 \le x \le 80$
$A'(x) = 120 - 3x = 0 \;\rightarrow\; x = 40, \qquad A(0) = 0, \quad A(40) = 2400, \quad A(80) = 0$
40 m by 60 m, with area 2400 m^2.

P03 (B) The sides of the base are $10 - 2x$ and $16 - 2x$; both must be at least 0, and $x \ge 0$.
$10 - 2x \ge 0$ gives $x \le 5$. So $0 \le x \le 5$. Answer B.

P04 $D(x) = (x - 2)^2 + (\sqrt{x})^2 = x^2 - 3x + 4, \qquad x \ge 0$
$D'(x) = 2x - 3 = 0 \;\rightarrow\; x = \frac{3}{2}$ (an upward parabola: the minimum)
The closest point is $\left(\frac{3}{2}, \sqrt{\frac{3}{2}}\right)$.

P05 $V'(x) = 12x^2 - 104x + 160 = 4(3x - 20)(x - 2)$
$x = \frac{20}{3} > 5$ is outside the domain. Candidates: $x = 0,\ 2,\ 5$.
$V(0) = 0, \quad V(2) = 2 \cdot 6 \cdot 12 = 144, \quad V(5) = 0$
The largest volume is 144 in^3, at $x = 2$.

P06 $A(x) = 2x\sqrt{4 - x^2}, \qquad 0 \le x \le 2$
$A'(x) = 2\sqrt{4 - x^2} - \dfrac{2x^2}{\sqrt{4 - x^2}} = \dfrac{8 - 4x^2}{\sqrt{4 - x^2}} = 0 \;\rightarrow\; x = \sqrt{2}$
$A(0) = 0, \quad A(\sqrt{2}) = 2\sqrt{2} \cdot \sqrt{2} = 4, \quad A(2) = 0$
The largest area is 4.