Lecture Notes · Unit 5 · Lecture 10 AP CED 5.10

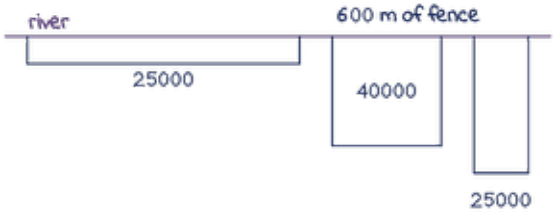
Introduction to Optimization

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: 600 METERS OF FENCE

Optimization: One Function, One Domain

AP Calculus 5.10



600 m fence, river side free

Biggest area?

$50 \cdot 500 = 25000$

$200 \cdot 200 = 40000$

$250 \cdot 100 = 25000$

Bet: Square?

SETTING IT UP: ONE VARIABLE

Name the parts → one function

2 short sides x, river side y

Maximize → $A = xy$

Stuck with → $2x + y = 600$

Two variables → one: $y = 600 - 2x$

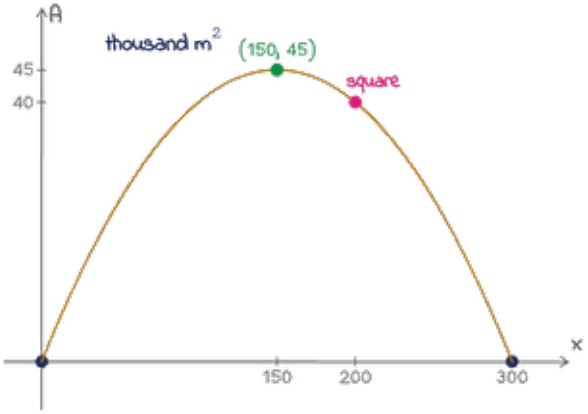
$A(x) = x(600 - 2x)$

$= 600x - 2x^2$

Objective $A(x)$ constraint $2x + y = 600$

DOMAIN FIRST, THEN THE DERIVATIVE

600 m of fence, river side free



$A(x) = x(600 - 2x)$

$x \geq 0,\ 600 - 2x \geq 0 \rightarrow 0 \leq x \leq 300$

$x = 400 \rightarrow y = -200$ X

$A'(x) = 600 - 4x = 0 \rightarrow x = 150$

$A(0) = 0,\ A(150) = 45000,\ A(300) = 0$

Square → 40000, top → 45000 m²

Not a square → $150 \times 300 = 45000$ m²

Ends stay on the list

FIVE STEPS FOR ANY OPTIMIZATION

Five steps

1 Picture → name the parts

2 Max or min → objective

3 Use the constraint → one variable

4 Domain: picture

5 Inside the domain → $f' = 0$ or no f'

Closed → compare the candidates

Two variables, one derivative?

✗ Common mistake

$A = xy \to A' = ?$

$(2x+y)'?$

✓ Do this

$y = 600 - 2x$

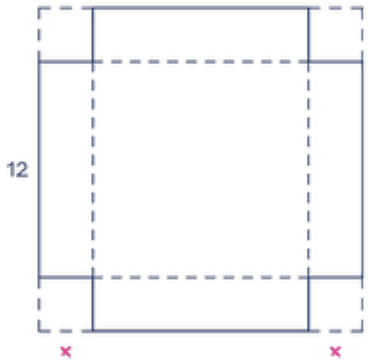
$A(x) \to A'(x)$

PROBLEM 1 OF 3

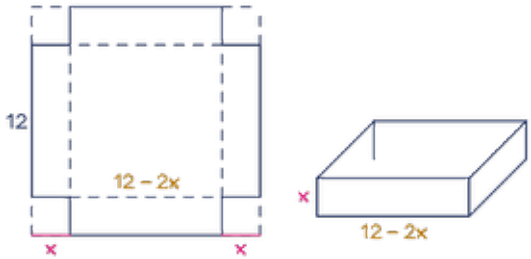
The open box

An open box is made from a 12 in by 12 in square sheet: cut an x by x square from each corner and fold up the sides.

What value of x gives the largest volume, and what is that volume?



Solution Set it up: fold the sheet



Plan: $V(x)$, domain $\to V' = 0$, compare

Fold up $\to$ height $= x$

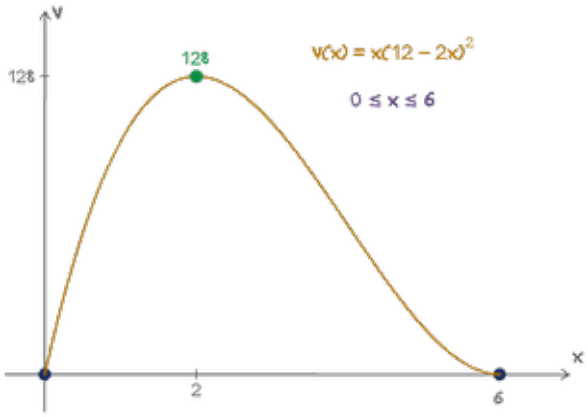
base $= 12 - x - x = 12 - 2x$

V = base area · height $\to V(x) = x(12-2x)^2$

$x \geq 0,\ 12 - 2x \geq 0 \to 0 \leq x \leq 6$

$V' = 0 \to$ compare

Solution Find the largest volume



Multiply out $\to V = 4x^3 - 48x^2 + 144x$

$V' = 12x^2 - 96x + 144$

$= 12(x-2)(x-6) \to x = 2, 6$

$x = 6 \to$ already an end

$V(0) = 0,\ V(2) = 2 \cdot 8^2 = 128,\ V(6) = 0$

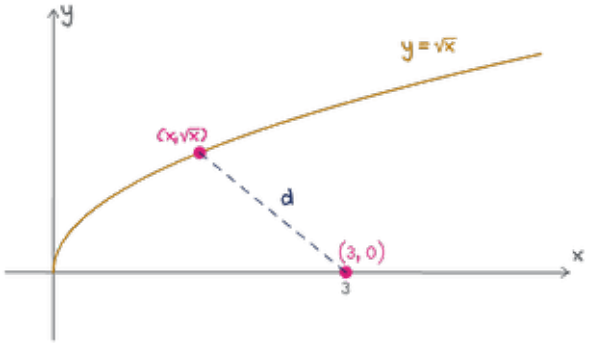
$\to x = 2$ in $\to V = 128$ in^3

Workbook → a 10 by 16 sheet

PROBLEM 2 OF 3

The closest point

Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.



Solution Set it up: distance squared

Plan: $D = d^2$ in terms of $x \rightarrow D'(x) = 0$

Constraint: $y = \sqrt{x} \rightarrow (x, \sqrt{x})$

distance formula

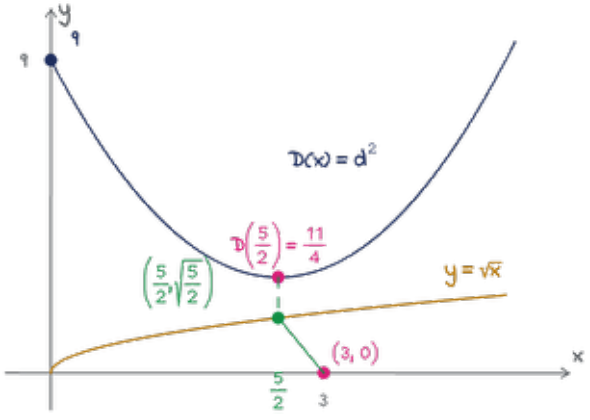
$\rightarrow d^2 = (x-3)^2 + (\sqrt{x} - 0)^2$

No root $\rightarrow d \geq 0 \rightarrow$ smallest $d \Leftrightarrow$ smallest d^2

Call $D(x) = d^2$

Your turn $\rightarrow$ simplify D, then D'

Solution Find the closest point



$D(x) = (x-3)^2 + x = x^2 - 5x + 9$

$x \geq 0$ (from $\sqrt{x}$)

$D'(x) = 2x - 5 = 0 \rightarrow x = \frac{5}{2}$

D opens up $\rightarrow$ lowest at vertex

$D(0) = 9$

$\frac{25}{4} - \frac{25}{2} + 9 = \frac{11}{4}$

$D = d^2 \rightarrow d = \frac{\sqrt{11}}{2}$

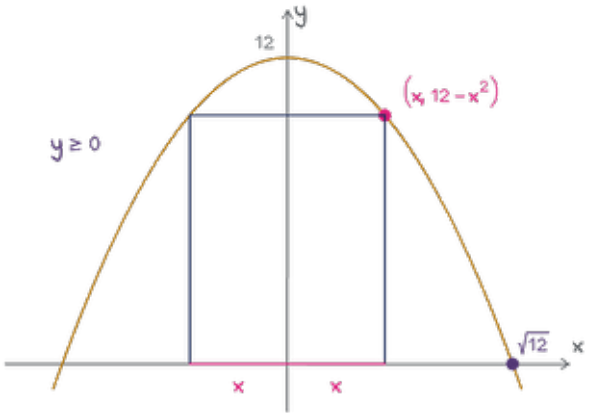
Closest point $\rightarrow \left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$

Workbook $\rightarrow$ closest to $(2, 0)$

PROBLEM 3 OF 3

The largest rectangle

A rectangle sits on the x-axis, with its two top corners on $y = 12 - x^2$. What is its largest possible area?



Solution Set it up: the corner on the curve

Plan: $A(x)$, a closed domain $\rightarrow$ compare

$(x, 12 - x^2)$ on $y = 12 - x^2$

$x + x = 2x$

height $= 12 - x^2$

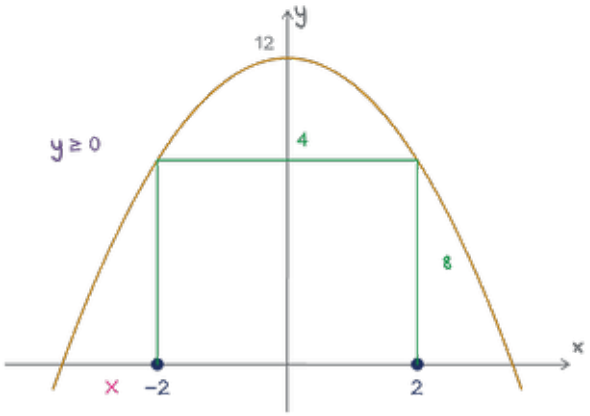
$A(x) = 2x(12 - x^2) = 24x - 2x^3$

$x = 0 \rightarrow$ no width; $x = \sqrt{12} \rightarrow$ no height

$\rightarrow 0 \le x \le \sqrt{12}$

Your turn $\rightarrow A' = 0$, then compare

Solution Find the largest area



$A' = 24 - 6x^2 = 0$

$x^2 = 4 \rightarrow x = \pm 2$

$-2 \rightarrow$ outside the domain X

$A(0) = 0, A(2) = 32, A(\sqrt{12}) = 0$

max area 32 (4 wide, 8 tall)

Workbook semicircle

WRAP-UP

One function, one domain

No → objective, constraint, domain, compare

outside the domain → no vote