5.10 / Introduction to Optimization Problems

Watch the lesson ➤

Setting up a function and its domain

Learning goal

Set up an optimization problem as one function of one variable with its domain, then find the largest or smallest value, ends included.

Key ideas

01 600 m of fence, one side along a river: $2x + y = 600$, so $y = 600 - 2x$ and $A(x) = x(600 - 2x) = 600x - 2x^2$.

02 The domain: $x \geq 0$ and $600 - 2x \geq 0$, so $0 \leq x \leq 300$. $x = 400$ would need $y = -200$: no fence.

03 $A'(x) = 600 - 4x = 0$ at $x = 150$. The ends are candidates too: $A(0) = 0$, $A(300) = 0$, $A(150) = 45{,}000$, the **biggest** area.

04 Five steps: 1. picture and names; 2. the constraint gives one variable; 3. the domain; 4. derivative = 0, compare with the ends; 5. answer the question, with units.

Remember

constraint
The fixed amount (fence, sum, sheet) that ties the variables together.

domain
The x values that make sense; its ends are candidates.

Class notes

5.10 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 No domain, no comparing

✗ $V' = 0$ at 2 and 6, so take $x = 2$.

✓ Domain $0 \le x \le 6$: candidates 0, 2, 6. Compare V: 0, 128, 0.

5.10 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Set it up, closed domain	One variable, the domain from the picture, derivative = 0, then compare with the ends.	E01, E03 · Practice 1, 2, 3, 5, 6
Distance	Use $D = d^2$: smaller d means smaller d^2.	E02 · Practice 4

Example 01

Video · E01

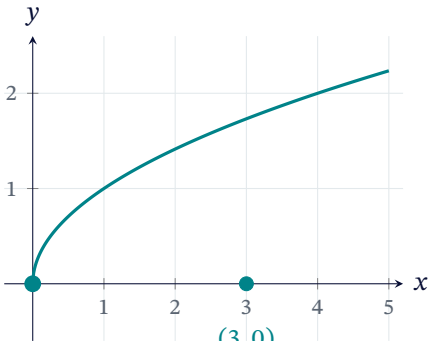
A 12 in by 12 in square sheet. Cut an x by x square from each corner and fold up the sides into an open box.
What x gives the largest volume? What is it?

> Volume as one function of x, its domain, then compare $V' = 0$ with the ends.

Example 02

Video · E02

Find the point on the curve $y = \sqrt{x}$ that is closest to the point $(3, 0)$.

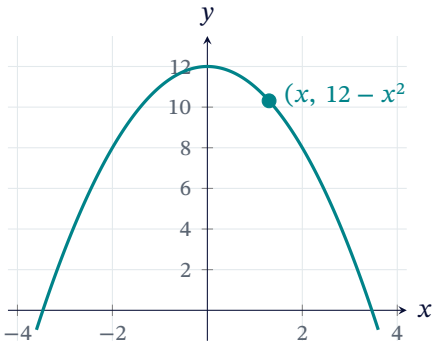


> A smaller distance means a smaller distance squared: minimize $D = d^2$.

Example 03

Video · E03

A rectangle sits on the x-axis,
with its two top corners on $y = 12 - x^2$.
What is its largest possible area?



Plan: $A(x) \rightarrow$ candidates

VIDEO COMPLETE Check your example answers at the end of this booklet.

5.10 / Example answers

Introduction to Optimization Problems

E01 $V(x) = x(12-2x)^2, \qquad 0 \le x \le 6$
$V'(x) = 12x^2 - 96x + 144 = 12(x-2)(x-6) = 0 \;\to\; x = 2,\ 6$
$V(0) = 0, \quad V(2) = 2 \cdot 8^2 = 128, \quad V(6) = 0$
The largest volume is 128 in^3, at $x = 2$ in.

E02 A point on the curve: $(x, \sqrt{x})$. $d^2 = (x-3)^2 + (\sqrt{x})^2$.
$D(x) = x^2 - 6x + 9 + x = x^2 - 5x + 9, \qquad x \ge 0$
$D'(x) = 2x - 5 = 0 \;\to\; x = \frac{5}{2}$ (an upward parabola: the minimum)
The closest point is $\left(\frac{5}{2}, \sqrt{\frac{5}{2}}\right)$, at distance $\sqrt{\frac{11}{4}} = \frac{\sqrt{11}}{2}$.

E03 Width $2x$, height $12 - x^2$; the domain $0 \le x \le \sqrt{12}$ is closed.
$A(x) = 2x(12 - x^2) = 24x - 2x^3, \quad A'(x) = 24 - 6x^2 = 0 \;\to\; x = 2$
Candidates: $A(0) = 0, A(\sqrt{12}) = 0, A(2) = 4 \cdot 8 = 32$. The largest area is 32.