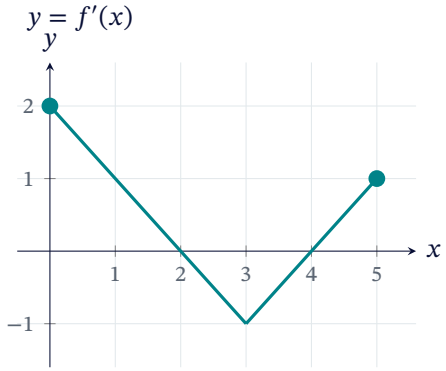
5.9 / Practice workbook

Connecting f, f', and f"

Practice

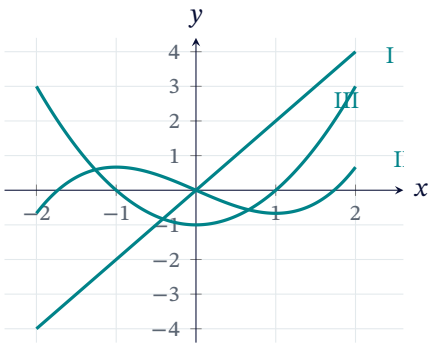
Problems 1–6 / Show your reasoning

1 The graph of f', the derivative of f, is shown for $0 \le x \le 5$. On what intervals is f increasing? Where does f have a relative maximum? Justify your answer.



2 The function f is twice differentiable, with $f'(2) = -3$ and $f''(2) = 4$. At $x = 2$, is f increasing or decreasing? Is the graph of f concave up or concave down? Give a reason for each answer.

3 The graphs of f, f', and f'' are shown, labeled I, II, and III. Which of the following is correct?



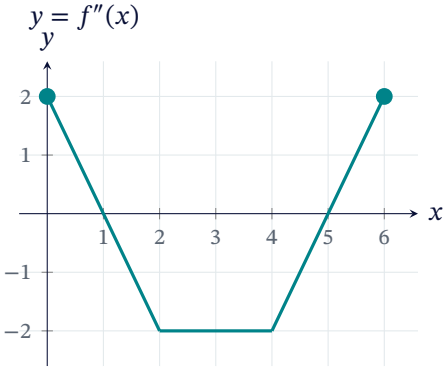
A. f: I, f': II, f'': III
B. f: III, f': I, f'': II
C. f: I, f': III, f'': II
D. f: II, f': III, f'': I

4 The function f is twice differentiable. Selected values of f, f', and f'' are shown in the table. Which of the following must be true?

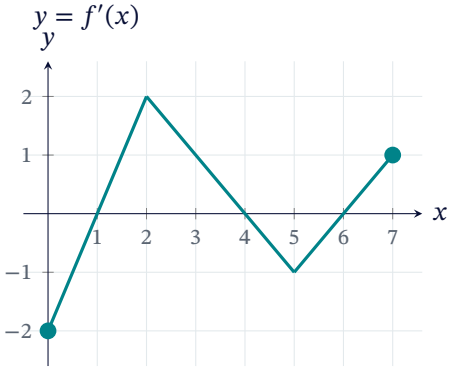
x	0	1	2	3
$f(x)$	4	2	1	3
$f'(x)$	-3	-1	0	2
$f''(x)$	2	2	1	0

A. f has a relative minimum at $x = 2$
B. f has a point of inflection at $x = 3$
C. f has a relative maximum at $x = 1$
D. f is decreasing at $x = 3$

5 The graph of f'', the second derivative of f, is shown for $0 \le x \le 6$. On what interval is f concave down? At what value of x does f' have a relative maximum? Justify your answer.



6 The graph of f', the derivative of f, is shown for $0 \le x \le 7$. On what intervals is f decreasing and concave up? Find all x where f has a point of inflection. Justify your answer.



5.9 / Answer key

Connecting f, f', and f"

P01 $f' > 0$ on $(0, 2)$ and $(4, 5)$, so f is increasing there.
f' changes from $+$ to $-$ at $x = 2$, so f has a relative maximum at $x = 2$.

P02 $f'(2) = -3 < 0$, so f is decreasing at $x = 2$.
$f''(2) = 4 > 0$, so the graph of f is concave up at $x = 2$.

P03 (D) II is flat at -1 and 1, and III is 0 there: III is the slope of II.
III is flat at 0, and I is 0 there: I is the slope of III. So f: II, f': III, f'': I. Answer D.

P04 (A) $f'(2) = 0$ and $f''(2) = 1 > 0$: a relative min at $x = 2$. (A) must be true.
Not (B): $f''(3) = 0$ alone. Not (C): $f'(1) = -1 \neq 0$. Not (D): $f'(3) = 2 > 0$. Answer A.

P05 $f'' < 0$ on $(1, 5)$, so f is concave down on $(1, 5)$.
f'' is the derivative of f'. f'' changes from $+$ to $-$ at $x = 1$, so f' has a relative maximum at $x = 1$.

P06 $f' < 0$ on $(0, 1)$ and $(4, 6)$: decreasing. f' is increasing on $(0, 2)$ and $(5, 7)$: concave up.
Both: $(0, 1)$ and $(5, 6)$.
f' changes from increasing to decreasing at $x = 2$ and from decreasing to increasing at $x = 5$: points of inflection at $x = 2$ and $x = 5$.