Lecture Notes · Unit 5 · Lecture 9 AP CED 5.9

Connecting f, f', and f''

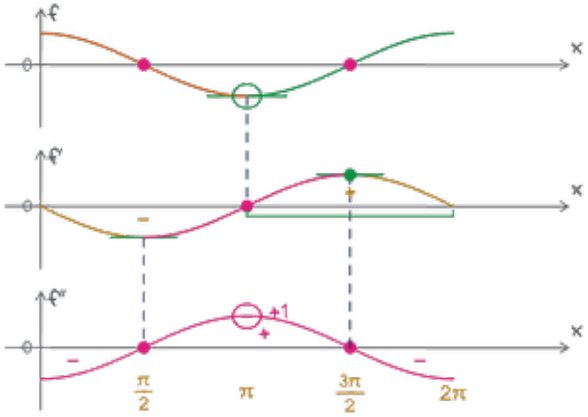
Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

WHOSE GRAPH IS IT? THE CHAIN OF SLOPES

Connecting f, f', f'' AP Calculus 5.9

one graph → whose? f, f', or f''?

Goal → tell them apart, name every reason



Slopes $f = \cos x$ $[0, 2\pi]$

$f' = -\sin x \rightarrow f'(\pi) = 0$

f' flat valley $\frac{\pi}{2}$ peak $\frac{3\pi}{2}$

$f'' = -\cos x \rightarrow 0$ at $\frac{\pi}{2}, \frac{3\pi}{2}$

Zero on one → flat on the one above

Read all three, down one x

$f' > 0$ on $(\pi, 2\pi) \rightarrow$ f increasing

$f' < 0$ on $(0, \pi) \rightarrow$ f decreasing

f' − to + at $\pi \rightarrow$ rel. min of f

2nd derivative test $f''(\pi) = 1 > 0$

$\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ $f'' > 0 \rightarrow$ f concave up

Ends: $f'' < 0 \rightarrow$ concave down

$\frac{\pi}{2}, \frac{3\pi}{2} \rightarrow$ f' valley, peak → inflection

one f' graph → 3 questions about f

1. $f' > 0$ $(\pi, 2\pi) \rightarrow$ f increasing

2. f' increasing $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right) \rightarrow$ f concave up

3. $f'\left(\frac{3\pi}{2}\right) = 1 \rightarrow$ f rises fastest (ends: 0)

Height of f' ≠ rise of f'

Reasons → whose graph?

Rel. max of f → f': + → −

Inflection: f' ↑↓ or ↓↑ = f' rel. extremum

2 properties → 2 reasons

e.g. $f' < 0$ and f' decreasing

Given f'' → reasons about f'

Name it → f', not 'it', not 'the graph'

x(t) → v(t) → a(t) → same chain

PROBLEM 1 OF 3

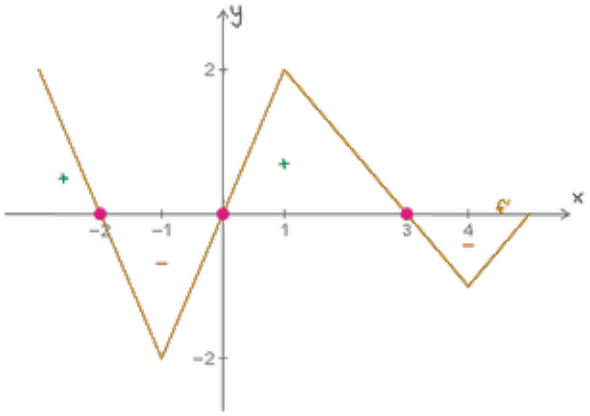
Reading the graph of f'

f is continuous on $[-3, 5]$. The graph of f' is shown. Justify each answer.

(a) Where does f have a relative maximum?

(b) Where does f have a point of inflection?

(c) On what open intervals is f decreasing and concave down?



Solution (a) Relative maximum

(a)

Given f' → signs first

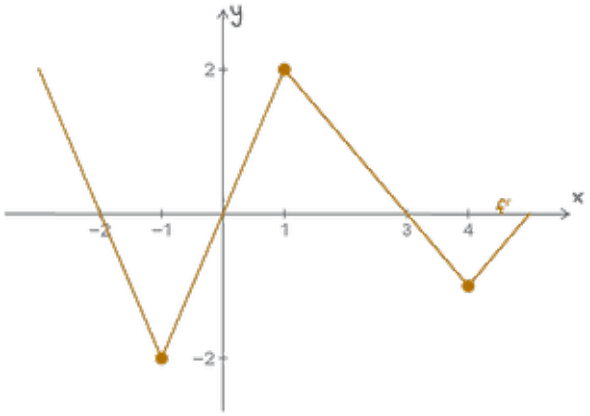
f' = 0 at x = −2, 0, 3

−2, 3 f': + → −

→ rel. max of f at x = −2, 3

x = 0 → − to + → min, not asked

Solution (b) Points of inflection



(b)

inflection → where f' turns

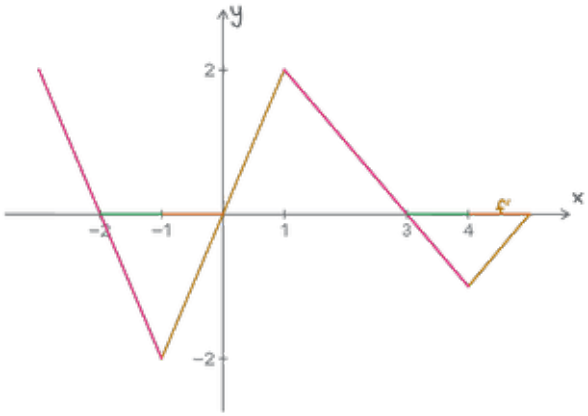
−1, 4 → f' falling then rising

f' inc → dec at x = 1

Corners → still rel. extrema of f'

inflection x = −1, 1, 4

Solution (c) Decreasing and concave down



(c)

2 properties → 2 reasons

decreasing → f' < 0 → (−2, 0), (3, 5)

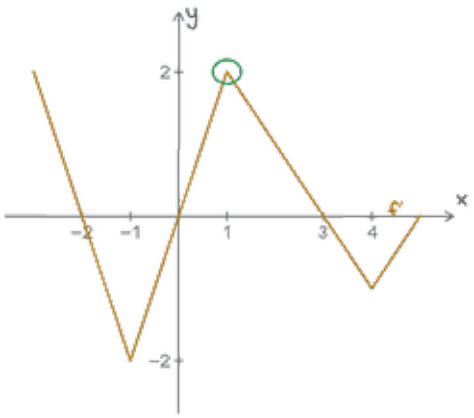
Concave down → f' falls → (−3, −1), (1, 4)

(−2, −1) (3, 4)

because f' < 0 and f' decreasing

Workbook → another f' graph

Lost reason



✗ Common mistake

f: bend flips → definition

Graph ↑↓ → which graph?

✓ Do this

f' rises, falls $x = 1$

2025 Q4(b) → whose graph?

PROBLEM 2 OF 3

A table of f, f', f''

f is twice differentiable. Which must be true?

(A) f has a relative max at $x = 2$

(B) f has a relative min at $x = 3$

(C) f has an inflection point at $x = 2$

(D) f is increasing on $(1, 3)$

x	1	2	3
$f(x)$	5	3	1
$f'(x)$	0	−2	0
$f''(x)$	−2	0	3

Solution One choice at a time

$f' = 0 \rightarrow f''$

$f'(3) = 0$ $f''(3) = 3 > 0$

rel. min at 3 → (B)

(A) $f'(2) = -2 \neq 0$ ✗

(C) $f''(2) = 0$ alone → sign change needed → ✗

(D) $f(3) = 1 < f(1) = 5$ → not increasing → ✗

Table → points only, not intervals

So (B)

Workbook → one column at a time

PROBLEM 3 OF 3

Which is f, f', f''?

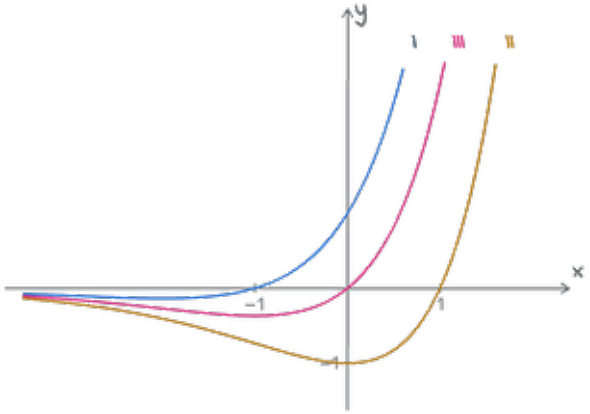
The graphs of f, f', f'' are labeled I, II, III. Which is correct?

(A) f: I, f': II, f'': III

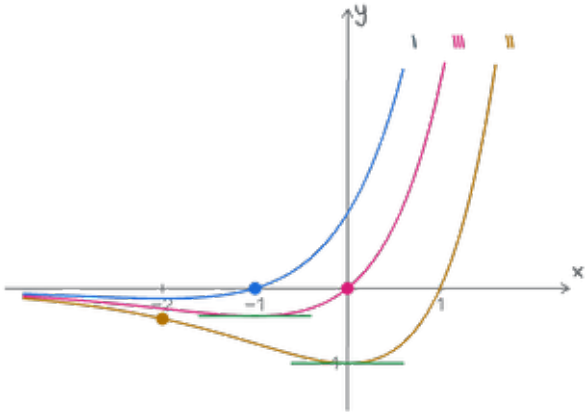
(B) f: II, f': I, f'': III

(C) f: II, f': III, f'': I

(D) f: III, f': II, f'': I



Solution Follow the chain



Plan → derivative zero ⇔ flat

II flat at 0 · III crosses 0

→ III = slope of II

III flat at −1 · I crosses 0

→ I = slope of III

II → III → I → $f = II$, $f' = III$, $f'' = I$

(A) I at −2: flat $II \neq 0$ ✗

(B) II flat at 0, $I \neq 0$ → ✗

(D) III flat at −1, $II \neq 0$ → ✗

Workbook → chain first

WRAP-UP

Whose graph is it?

Sign of f' → up or down; rise of f' → bend

Name the graph; one reason per property

Next → 5.10 optimization

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.