5.9 / Connecting f, f', and f"

Watch the lesson ➤

Reasons about the given graph

Learning goal
Connect f, f', and f'': read each property from the right graph or table, and give each property its own reason about the given graph.

Key ideas

01 A chain of slopes: $f = \cos x$, $f' = -\sin x$, $f'' = -\cos x$. Each graph is the slope graph of the one above: zero below means a flat spot above.

02 One table: $f' > 0$ increasing, $f' < 0$ decreasing; f' changes sign: a relative extremum; $f'(c) = 0$, $f''(c) > 0$: relative min. $f'' > 0$: f' increasing, concave up; $f'' < 0$: concave down.

03 Inflection: f' has a peak or valley, or f'' changes sign.

04 Say it about the given graph. Given f': "f' changes from + to −" (relative max); "f' changes from increasing to decreasing" (inflection).

05 Two properties, two reasons: "$f' < 0$ and f' is decreasing." Name f', not "it."

Remember

the given graph
Write the reason about the graph shown (f' or f''), not about "the graph."

Class notes

5.9 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 The wrong graph in the reason

✗ f changes concavity at 1 because the graph changes direction.

✓ Say it about the given graph: f' changes from increasing to decreasing at $x = 1$.

5.9 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

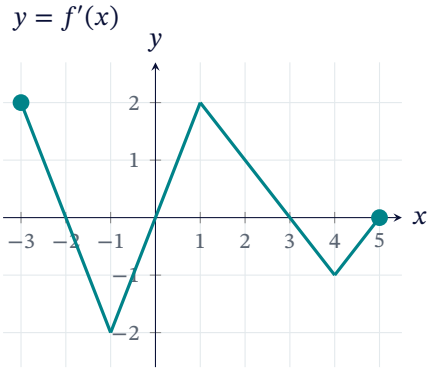
Read the problem, name the type, then use its method

Type	Method	Problems
Reasons about the given graph	Say each property with the graph that was given: the sign of f', or f' increasing or decreasing.	E01 · Practice 1, 5, 6
A table of f, f', f"	Read each value: f' for up or down, f'' for the bend.	E02 · Practice 2, 4
Which graph is which	Zero below = flat above: find the chain.	E03 · Practice 3

Example 01

Video · E01

f is continuous on $[-3, 5]$. The graph of f' is shown. Justify each.
(a) Where does f have a relative maximum?
(b) Where does f have a point of inflection?
(c) On what open intervals is f decreasing and concave down?



> Every reason is about f': its sign, and whether it rises or falls.

Example 02

Video · E02

f is twice differentiable. Which must be true?

x	1	2	3
$f(x)$	5	3	1
$f'(x)$	0	-2	0
$f''(x)$	-2	0	3

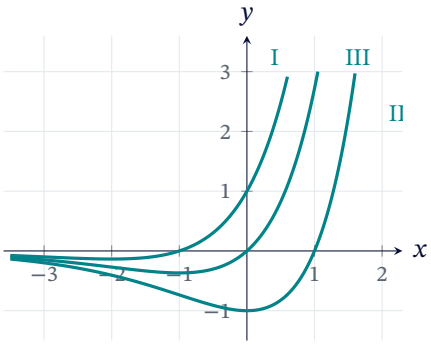
A. relative max at $x = 2$
B. relative min at $x = 3$
C. point of inflection at $x = 2$
D. f increasing on $(1, 3)$

> $f'(c) = 0$ first, then the sign of $f''(c)$.

Example 03

Video · E03

The graphs of f, f', f'' are labeled I, II, III. Which is correct?



A. f: I, f': II, f'': III
B. f: II, f': I, f'': III
C. f: II, f': III, f'': I
D. f: III, f': II, f'': I

> Zero below means flat above.

VIDEO COMPLETE Check your example answers at the end of this booklet.

5.9 / Example answers

Connecting f, f', and f"

E01 (a) f' changes from + to − at $x = -2$ and $x = 3$: relative maxima there.
(b) f' changes from decreasing to increasing at $x = -1$ and $x = 4$, and from increasing to decreasing at $x = 1$: points of inflection at $x = -1,\ 1,\ 4$.
(c) $f' < 0$ on $(-2, 0)$ and $(3, 5)$; f' is decreasing on $(-3, -1)$ and $(1, 4)$.
Both: $(-2, -1)$ and $(3, 4)$, where $f' < 0$ and f' is decreasing.

E02 (B) $f'(3) = 0$ and $f''(3) = 3 > 0$: a relative min at $x = 3$. (B) must be true.
Not (A): $f'(2) = -2 \neq 0$. Not (C): $f''(2) = 0$ alone. Not (D): $f(3) < f(1)$. Answer B.

E03 (C) II is flat at 0, and III is 0 there: III is the slope of II.
III is flat at -1, and I is 0 there: I is the slope of III.
So f: II, f': III, f'': I. Answer C.