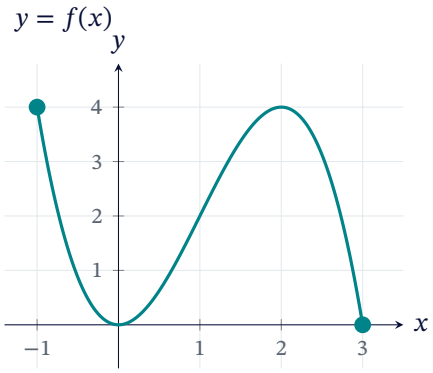
# 5.8 / Practice workbook

*Sketching Graphs of f and f'*

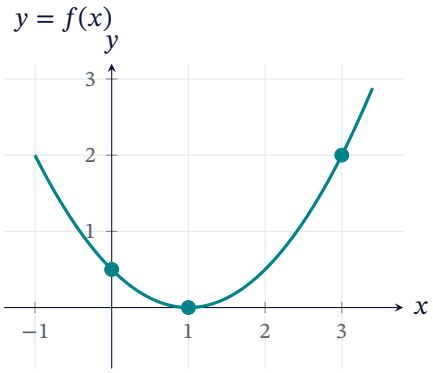
## Practice

Problems 1–7 / Show your reasoning

1 The graph of $f$ is shown for $-1 \le x \le 3$. Where is $f'(x) = 0$? On what intervals is $f'(x) > 0$, and on what intervals is $f'(x) < 0$?



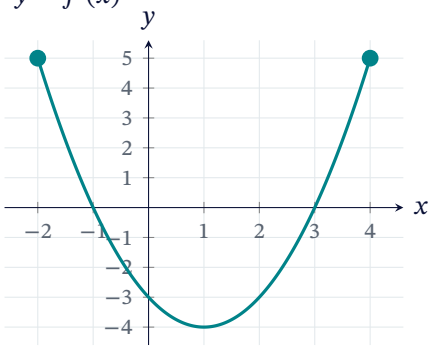

2 The graph of $f$ is shown, with points at $x = 0$, 1, and 3. Which of the following is true?




A. $f'(0) < f'(1) < f'(3)$
B. $f'(1) < f'(0) < f'(3)$
C. $f'(3) < f'(1) < f'(0)$
D. $f'(1) < f'(3) < f'(0)$

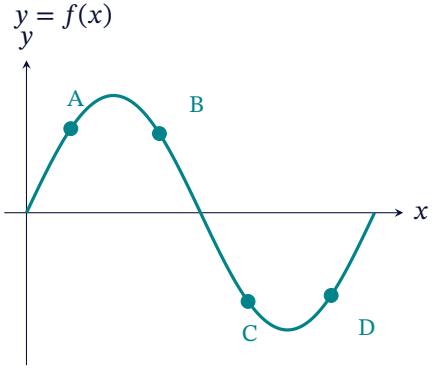
3 The graph of $f'$, the derivative of $f$, is shown for $-2 \le x \le 4$. Find all $x$ where $f$ has a relative extremum, and classify each. On what interval is $f$ concave up? Justify your answer.

$y = f'(x)$




4 The graph of $f$ is shown, with points $A$, $B$, $C$, and $D$. At which point is $f'(x) > 0$ and $f''(x) < 0$?

$y = f(x)$

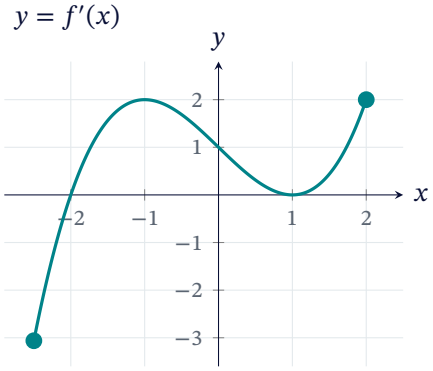



A. $A$
B. $B$
C. $C$
D. $D$

5 The function $f$ is twice differentiable on $[0, 6]$, with $f(0) = 0$, $f(2) = 2$, $f(4) = 1$, and $f(6) = 0$. Also $f' > 0$ on $(0, 2)$, $f' < 0$ on $(2, 6)$, $f'' < 0$ on $(0, 4)$, and $f'' > 0$ on $(4, 6)$. Sketch a possible graph of $f$. Where does $f$ have a relative maximum, and where does it have a point of inflection?

6 The graph of $f'$, the derivative of $f$, is shown for $-2.5 \leq x \leq 2$. Which of the following is true?

$y = f'(x)$




A. $f$ has a relative maximum at $x = 1$
B. $f$ is decreasing on $(-2, 1)$
C. $f$ is concave down on $(-1, 1)$
D. $f$ has a relative maximum at $x = -2$

## 5.8 / AP-style free response

*Problem 7* ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = x^4 - 4x^3$.

**(a)** Find $f'(x)$ and $f''(x)$ in factored form.

**(b)** On what intervals is $f$ increasing, and on what intervals is it decreasing? On what intervals is the graph of $f$ concave up, and on what intervals is it concave down?

**(c)** Find the relative extrema and the points of inflection of the graph of $f$. Then sketch the graph of $f$.

# 5.8 / Answer key

*Sketching Graphs of f and f'*

**1** $f' = 0$ at $x = 0,\ 2$; $f' > 0$ on $(0, 2)$; $f' < 0$ on $(-1, 0)$ and $(2, 3)$

**2** (A)

**3** relative max at $x = -1$, relative min at $x = 3$; concave up on $(1, 4)$

**4** (A)

**5** relative max at $x = 2$; point of inflection at $x = 4$

**6** (C)

**7** (a) $f'(x) = 4x^2(x - 3)$, $f''(x) = 12x(x - 2)$; (b) $f'$: $-,\ -,\ +$; $f''$: $+,\ -,\ +$; (c) min $(3, -27)$; inflection $(0, 0)$ and $(2, -16)$; $x = 0$ is flat but not an extremum