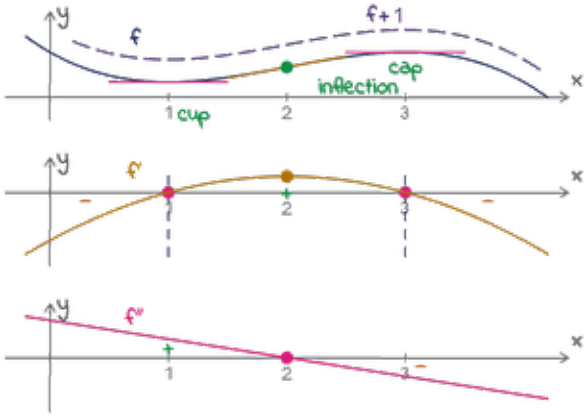
Lecture Notes · Unit 5 · Lecture 8 AP CED 5.8

Sketching f and f′

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO AND IDEA 1: FROM F TO ITS SLOPES

From f to f′, slope by slope AP Calculus 5.8



Sketch f′ from f, then f from f′

f → f′?

f → flat at 1, 3 · steepest at 2

f flat → f′ = 0 (x = 1, 3)

f rises → f′ above 0 · f falls → below 0

Steepest at 2 → f′ highest

Corner in f → f′ jumps (no f′ there)

From f′ back to f

f′ → −, +, − · f rises (+) · f falls (−)

− to + at 1 → valley · + to − at 3 → peak

f′ rising (x < 2) → cup

f′ falling (x > 2) → cap

At 2 → bend of f flips · f′ peak (≠ 0)

$f' = -x^2 + 4x - 3 \rightarrow f'' = 4 - 2x$

f″ → + | 0 (x = 2) | − → cup to cap

f′ → shape · f height?

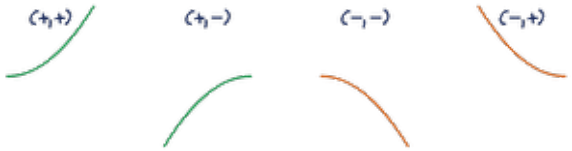
$f(x) = -\frac{1}{3}x^3 + 2x^2 - 3x + 2$. Another shift: f(x) + 5

1 point of f → height

IDEA 3: FOUR PIECES

Signs of f′ and f″ → the piece

Signs in order: (f′, f″)



f → rising cup

rising cap

falling cap

falling cup

1 interval → 1 piece

Join. 1 point → height

PROBLEM 1 OF 3

Order the slopes

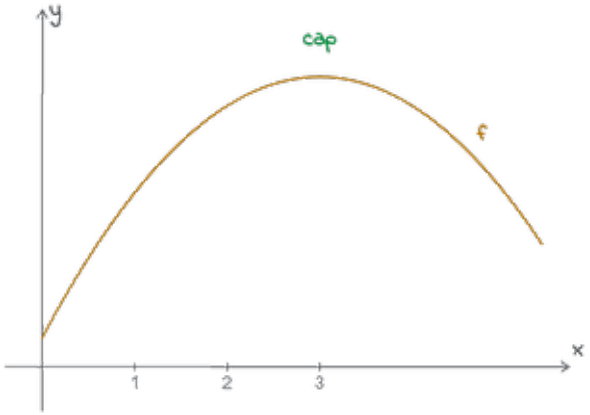
The graph of f is shown. Which is true?

(A) $f'(1) < f'(2) < f'(3)$

(B) $f'(3) < f'(2) < f'(1)$

(C) $f'(2) < f'(3) < f'(1)$

(D) $f'(3) < f'(1) < f'(2)$



Solution Bend → f'' → f'

This is f · asks f′ → not heights

Plan → bend → f″ → f′

Cap → f″ < 0

→ f′ decreasing

1 < 2 < 3 → f′(3) < f′(2) < f′(1) (B)

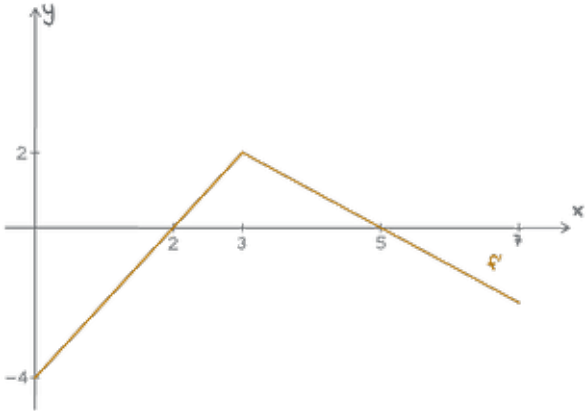
Workbook → rank slopes

PROBLEM 2 OF 3

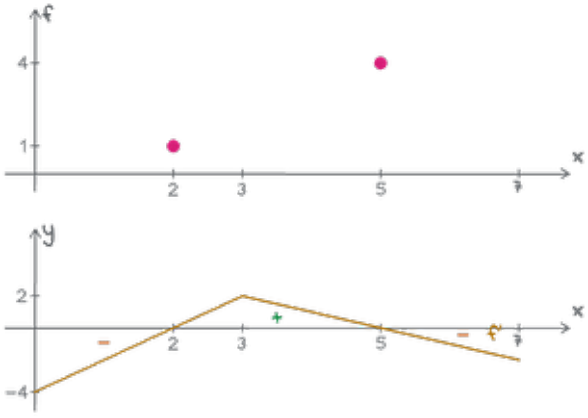
From f' to f

The graph of f' is shown for $0 \le x \le 7$, with $f(2) = 1$ and $f(5) = 4$.

Sketch the graph of f.



Solution Signs of f'



Graph = f′ (not f)

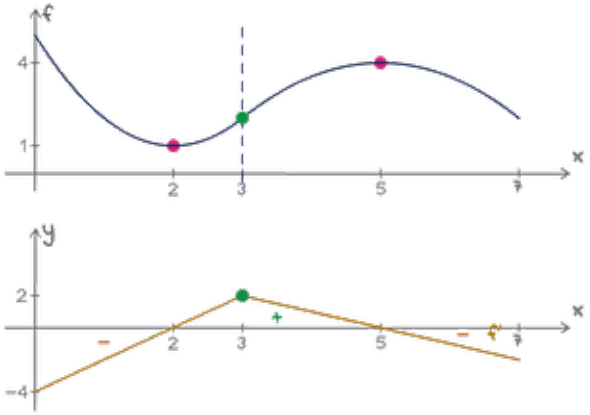
(0, 2) → f falls

(2, 5) → f rises

(5, 7) → f falls

valley (2, 1) · peak (5, 4)

Solution The bend, then draw



The bend of f

(0, 3) → f′ rising → cup

(3, 7) → f′ falling → cap

x = 3 → no peak → bend flips

f′ corner → one slope: f′(3) = 2

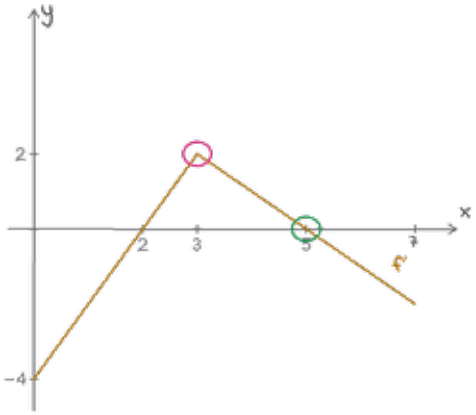
cup down to (2, 1) up to 3

cap up to (5, 4) down to 7

x = 3: inflection of f → peak of f′

Workbook → read it as f′

f′ or f?



✗ Common mistake

f′ peak at 3 → f peak?

f′ ↓ → f ↓?

✓ Do this

2 < x < 5 → f ↑

f′ ↓ → cap

2, 5 → f′ crosses 0

f′

PROBLEM 3 OF 3

From signs to a sketch

f is twice differentiable on $[0, 6]$. $f(1) = 5$, $f(3) = 3$, $f(5) = 1$.

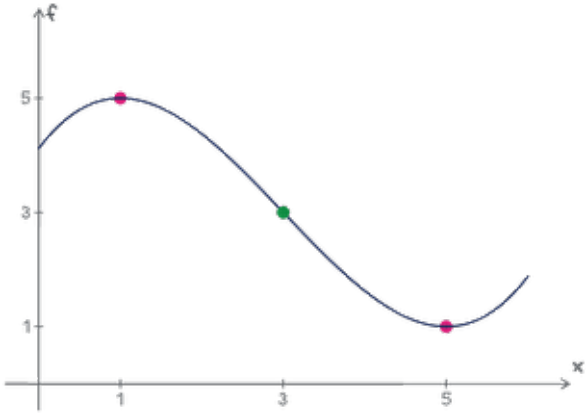
On $(0, 1) \rightarrow f' > 0,\ f'' < 0$

On $(1, 3) \rightarrow f' < 0,\ f'' < 0$

On $(3, 5) \rightarrow f' < 0,\ f'' > 0$

On $(5, 6) \rightarrow f' > 0,\ f'' > 0$

Sketch the graph of f.



Solution One piece per interval

2 steps → points → pieces

(1, 5) (3, 3) (5, 1)

(0, 1): +, − → cap (1, 5)

(1, 3) → −, − → falling cap

(3, 5) → −, + → falling cup

(3, 3) → inflection

(5, 6) → +, + → rising cup (5, 1)

Peak · inflection · valley

Workbook → piece by piece

WRAP-UP

Draw f′ from f, and back

f′ crosses 0 → peak or valley of f · f′ turns → bend of f flips

f′ → shape · a point → height

Signs of f′, f″ → one of four pieces

Which graph? → check the label

Next → 5.9: read f, f′, f″ and write the reasons

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.