5.8 / Sketching Graphs of f and f'

Watch the lesson ➤

Reading one graph from the other

Learning goal
Go from the graph of f to the graph of f' and back, and read rising, falling, cups, and caps from the signs of f' and f''.

Key ideas

01 From f to f': where f is flat, $f' = 0$; rising, $f' > 0$ (above the axis); falling, $f' < 0$ (below). Steeper means $|f'|$ is bigger.

02 From f' to f: $f' > 0$, f rises; $f' < 0$, f falls. f' changes $-$ to $+$: a valley; $+$ to $-$: a peak.

03 f' rising ($f'' > 0$): a cup. f' falling ($f'' < 0$): a cap. Where f' changes from rising to falling, the bend changes.

04 Four pieces: $f' > 0$, $f'' > 0$ rising cup; $f' > 0$, $f'' < 0$ rising cap; $f' < 0$, $f'' < 0$ falling cap; $f' < 0$, $f'' > 0$ falling cup.

05 Which graph is shown? Read **that** one: the top of f' is not a peak of f.

Remember

cup / cap
Concave up ($f'' > 0$) / concave down ($f'' < 0$).

graph of f'
Its height is the slope of f.

Class notes

5.8 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Reading f' like f

✗ f' has its top at 3, so f has a maximum at 3. f' falls, so f falls.

✓ Peaks and valleys of f are where f' crosses 0. f' falling means f is a cap.

5.8 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

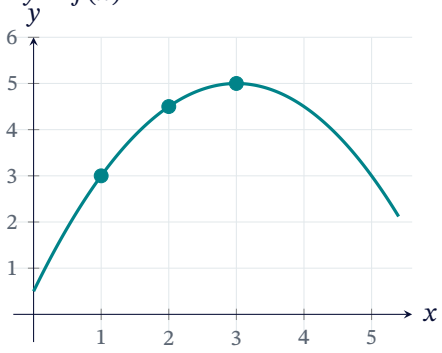
Type	Method	Problems
From f to f’	Slope by slope: flat → 0, rising → above, falling → below.	E01 · Practice 1, 2
From f’ to f	Signs of f' → rises or falls; f' rising or falling → cup or cap.	E02 · Practice 3, 6, 7
Four pieces	The signs of f' and f'' on each interval → one of four pieces.	E03 · Practice 4, 5

Example 01

Video · E01

The graph of f is shown. Which is true?

$y = f(x)$



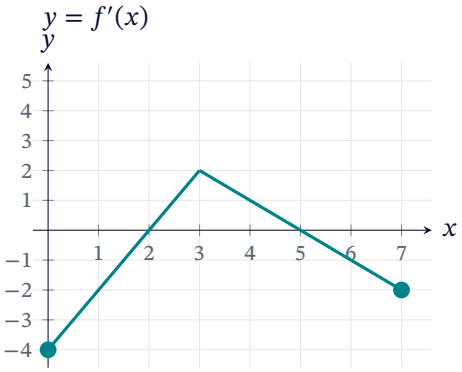
A. $f'(1) < f'(2) < f'(3)$
B. $f'(3) < f'(2) < f'(1)$
C. $f'(2) < f'(3) < f'(1)$
D. $f'(3) < f'(1) < f'(2)$

> Read the slope at each point, not the height.

Example 02

Video · E02

The graph of f' is shown, $0 \le x \le 7$. $f(2) = 1$, $f(5) = 4$.
Sketch the graph of f.



> The sign of f': up or down. f' rising or falling: a cup or a cap.

Example 03

Video · E03

f is twice differentiable on $[0, 6]$. $f(1) = 5$, $f(3) = 3$, $f(5) = 1$.
On $(0, 1)$? $f' > 0$, $f'' < 0$
On $(1, 3)$? $f' < 0$, $f'' < 0$
On $(3, 5)$? $f' < 0$, $f'' > 0$
On $(5, 6)$? $f' > 0$, $f'' > 0$
Sketch the graph of f.

> Plot the three points, then one piece per interval.

VIDEO COMPLETE Check your example answers at the end of this booklet.

5.8 / Example answers

Sketching Graphs of f and f'

E01 (B)

E02 falls on $(0, 2)$, rises on $(2, 5)$, falls on $(5, 7)$; valley $(2, 1)$, peak $(5, 4)$; concave up on $(0, 3)$, down on $(3, 7)$

E03 rising cap to the peak $(1, 5)$; falling cap; bend changes at $(3, 3)$; falling cup to the valley $(5, 1)$; rising cup