5.7 / Practice workbook

The Second Derivative Test

Practice

Problems 1–6 / Show your reasoning

1 Let $f(x) = 2x^3 - 3x^2 - 12x$. Use the Second Derivative Test to find the relative extrema of f.

2 Let $f(x) = x^4 - 8x^2$. Use the Second Derivative Test to classify each critical point of f.

3 The function f is twice differentiable, with $f'(2) = 0$ and $f''(2) = 0$. Which of the following must be true?

A. f has a relative maximum at $x = 2$
B. f has a relative minimum at $x = 2$
C. f has a point of inflection at $x = 2$
D. None of these must be true

4 Let $g(x) = x + 2\cos x$ for $0 < x < 2\pi$. Use the Second Derivative Test to find where g has a relative maximum and a relative minimum.

5 Let $f(x) = x^2 + \frac{16}{x}$ for $x > 0$. Find the absolute minimum value of f for $x > 0$. Justify your answer.

6 The function f is twice differentiable. The table gives values of f' and f''. For each x in the table with $f'(x) = 0$, determine whether f has a relative maximum or a relative minimum, or whether the Second Derivative Test is inconclusive. Justify your answer.

x	-1	0	2	4
$f'(x)$	0	3	0	0
$f''(x)$	-2	0	5	0

5.7 / Answer key

The Second Derivative Test

P01 $f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1) \rightarrow x = -1,\ 2$
$f''(x) = 12x - 6, \qquad f''(-1) = -18 < 0, \qquad f''(2) = 18 > 0$
Relative max at $x = -1$; relative min at $x = 2$.

P02 $f'(x) = 4x^3 - 16x = 4x(x-2)(x+2) \rightarrow x = 0,\ \pm 2$
$f''(x) = 12x^2 - 16, \qquad f''(0) = -16 < 0, \qquad f''(\pm 2) = 32 > 0$
Relative max at $x = 0$; relative min at $x = -2$ and $x = 2$.

P03 (D) $(x-2)^4$ has $f'(2) = f''(2) = 0$ and a relative min, with no inflection: not A, not C.
$-(x-2)^4$ has a relative max: not B. None must be true. Answer D.

P04 $g'(x) = 1 - 2\sin x = 0 \rightarrow x = \frac{\pi}{6},\ \frac{5\pi}{6}$
$g''(x) = -2\cos x, \qquad g''\left(\frac{\pi}{6}\right) = -\sqrt{3} < 0, \qquad g''\left(\frac{5\pi}{6}\right) = \sqrt{3} > 0$
Relative max at $x = \frac{\pi}{6}$; relative min at $x = \frac{5\pi}{6}$.

P05 $f'(x) = 2x - \frac{16}{x^2} = 0 \rightarrow x^3 = 8 \rightarrow x = 2$
$f''(x) = 2 + \frac{32}{x^3}, \qquad f''(2) = 6 > 0, \qquad f(2) = 4 + 8 = 12$
f is continuous on $x > 0$, and $x = 2$ is its only critical point, a relative min. So the absolute minimum value is 12, at $x = 2$.

P06 $x = -1$: $f' = 0$, $f'' = -2 < 0$, a relative max. $x = 2$: $f' = 0$, $f'' = 5 > 0$, a relative min.
$x = 4$: $f' = 0$, $f'' = 0$, the test cannot tell.
$x = 0$: $f'(0) = 3 \neq 0$, not a critical point.