5.6 / Practice workbook

Concavity and Points of Inflection

Practice

Problems 1–7 / Show your reasoning

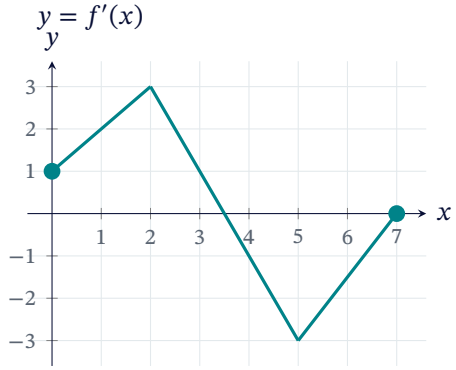
1 Let $f(x) = x^3 - 6x^2 + 5$. On what intervals is f concave up, and on what intervals is it concave down? Find the x-coordinate of the point of inflection. Justify your answer.

2 Let $g(x) = xe^x$. Find where g has a point of inflection. Justify your answer.

3 Let $f(x) = x^4 - 6x^2$. At what values of x does f have a point of inflection?

A. 0
B. ± 1
C. $\pm\sqrt{3}$
D. $0, \pm\sqrt{3}$

4 The graph of f', the derivative of f, is shown for $0 \le x \le 7$. On what interval is f concave down? Find all x where f has a point of inflection. Justify your answer.



5 Let $f(x) = \dfrac{x}{x^2+1}$, with $f''(x) = \dfrac{2x(x^2-3)}{(x^2+1)^3}$. On what intervals is f concave up? Find the x-coordinates of all points of inflection. Justify your answer.

6 Let $f(x) = e^{-x^2}$. Find the x-coordinates of all points of inflection of f. Justify your answer.

5.6 / AP-style free response

Problem 7 ➤ */ Show the work and the reasons a grader looks for.*

Let $f(x) = x^5 - 5x^4$.

(a) On what intervals is the graph of f concave up, and on what intervals is it concave down? Justify your answer.

(b) Find the x-coordinate of each point of inflection of the graph of f. Justify your answer.

5.6 / Answer key

Concavity and Points of Inflection

1 concave down on $(-\infty, 2)$, up on $(2, \infty)$; inflection at $x = 2$

2 $x = -2$

3 (B)

4 concave down on $(2, 5)$; inflection at $x = 2,\ 5$

5 concave up on $(-\sqrt{3}, 0)$ and $(\sqrt{3}, \infty)$; inflection at $x = -\sqrt{3},\ 0,\ \sqrt{3}$

6 $x = -\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}$

7 (a) $f''(x) = 20x^2(x - 3)$: $-,\ -,\ +$; down on $(-\infty, 0)$ and $(0, 3)$, up on $(3, \infty)$; (b) $x = 3$ only: f'' changes sign there; at $x = 0$, $f''(0) = 0$ but $f'' < 0$ on both sides