5.6 / Concavity and Points of Inflection

Watch the lesson ➤

Determining concavity of functions over their domains

Learning goal

Find where f is concave up or down from f'' (or from the graph of f'), find points of inflection, and give the reason that earns the point.

Key ideas

01 Two uphill curves: slopes 0.5, 1, 1.5 grow, so f' is increasing and the curve bends up: **concave up**. Slopes that shrink: f' decreasing, **concave down**.

02 $f'' > 0$ on an interval: f' increasing, concave up. $f'' < 0$: concave down.

03 A **point of inflection**: the concavity changes, so f'' changes sign. $f''(c) = 0$ is only a candidate.

04 $f(x) = x^3 - 3x$: $f''(x) = 6x$, concave down for $x < 0$, up for $x > 0$, inflection at 0, where f' has its minimum.

05 Write: "f'' changes from $-$ to $+$ at $x = c$, so f has a point of inflection there." Given the graph of f': "f' changes from decreasing to increasing."

Remember

concave up
f' is increasing ($f'' > 0$): the graph bends up.

concave down
f' is decreasing ($f'' < 0$): the graph bends down.

point of inflection
A point where f is continuous and the concavity changes (f'' changes sign).

Class notes

5.6 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 f” = 0 alone

✗ $f''(0) = 0$, so f has a point of inflection at 0.

✓ At 0, f'' stays negative: no point of inflection. The reason is that f'' changes sign.

5.6 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
The sign of f”	Zeros of f'', the sign on each piece, then the sentence.	E01, E02 · Practice 1, 2, 3, 5, 6, 7
From the graph of f’	Concave up where f' increases, down where it decreases; inflection where f' turns.	E03 · Practice 4

Example 01

Video · E01

Let $f(x) = 3x^5 - 5x^4$.
Find the intervals where f is concave up and concave down, and all x where f has a point of inflection. Justify your answer.

> Find where $f'' = 0$, then check whether f'' changes sign there.

Example 02

Video · E02

Let $f(x) = \ln(x^2 + 4)$. At what values of x does f have a point of inflection?

A. 0 only
B. ± 1
C. ± 2
D. ± 4

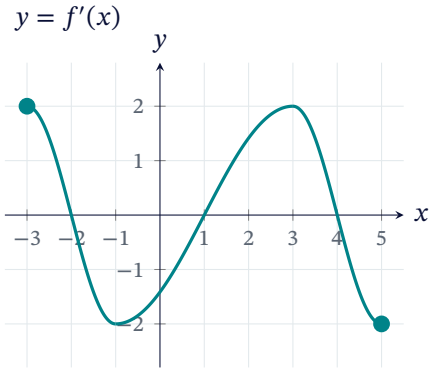
> The bottom of f'' is positive; the sign comes from the top.

Example 03

Video · E03

The graph of f', the derivative of f, is shown for $-3 \leq x \leq 5$. On which interval is the graph of f concave up?

Concave up: f' increasing. Not $f' > 0$.



A. $(-3, -1)$
B. $(-1, 3)$
C. $(1, 4)$
D. $(3, 5)$

VIDEO COMPLETE Check your example answers at the end of this booklet.

5.6 / Example answers

Concavity and Points of Inflection

E01 concave down on $(-\infty, 0)$ and $(0, 1)$, up on $(1, \infty)$; inflection at $x = 1$ only

E02 (C)

E03 (B)