5.5 / Practice workbook

The Candidates Test

Practice

Problems 1–6 / Show your reasoning

1 Find the absolute maximum and minimum values of $f(x) = x^3 - 3x$ on $[0, 2]$. Justify your answer.

2 Find the absolute maximum and minimum values of $f(x) = x^2 - 6x + 1$ on $[-1, 4]$. Justify your answer.

3 What is the absolute maximum value of $f(x) = x^4 - 2x^2$ on $[-2, 1]$?

A. 0
B. 1
C. 8
D. 16

4 Find the absolute maximum and minimum values of $f(x) = x - 2\sqrt{x}$ on $[0, 9]$. Justify your answer.

5 Find the absolute maximum and minimum values of $f(x) = xe^{-x}$ on $[0, 3]$. Justify your answer.

6 Find the absolute maximum and minimum values of $f(x) = x^{\frac{2}{3}}(5 - x)$ on $[-1, 5]$. Justify your answer.

5.5 / Answer key

The Candidates Test

P01 $f'(x) = 3x^2 - 3 = 3(x-1)(x+1) \;\to\; x = 1$ in $[0, 2]$
$f(0) = 0, \quad f(1) = -2, \quad f(2) = 2$
Absolute max 2 at $x = 2$; absolute min -2 at $x = 1$.

P02 $f'(x) = 2x - 6 = 0 \;\to\; x = 3$
$f(-1) = 8, \quad f(3) = -8, \quad f(4) = -7$
Absolute max 8 at $x = -1$; absolute min -8 at $x = 3$.

P03 (C) $f'(x) = 4x^3 - 4x = 4x(x-1)(x+1)$
$f(-2) = 8, \quad f(-1) = -1, \quad f(0) = 0, \quad f(1) = -1$
The largest is 8. Answer C.

P04 $f'(x) = 1 - \dfrac{1}{\sqrt{x}} = 0 \;\to\; x = 1$
$f(0) = 0, \quad f(1) = -1, \quad f(9) = 9 - 6 = 3$
Absolute max 3 at $x = 9$; absolute min -1 at $x = 1$.

P05 $f'(x) = e^{-x}(1-x) = 0 \;\to\; x = 1$
$f(0) = 0, \quad f(1) = \frac{1}{e}, \quad f(3) = \frac{3}{e^3} < \frac{1}{e} \;\; (3 < e^2)$
Absolute max $\frac{1}{e}$ at $x = 1$; absolute min 0 at $x = 0$.

P06 $f'(x) = \dfrac{5(2-x)}{3x^{\frac{1}{3}}}$: $f'(2) = 0$, $f'(0)$ DNE
$f(-1) = 6, \quad f(0) = 0, \quad f(2) = 3\sqrt[3]{4}, \quad f(5) = 0$
$\sqrt[3]{4} < 2$, so $3\sqrt[3]{4} < 6$. Absolute max 6 at $x = -1$; absolute min 0 at $x = 0$ and $x = 5$.