Lecture Notes · Unit 5 · Lecture 5 AP CED 5.5

The Candidates Test

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: THE HIGHEST POINT ON A TRAIL

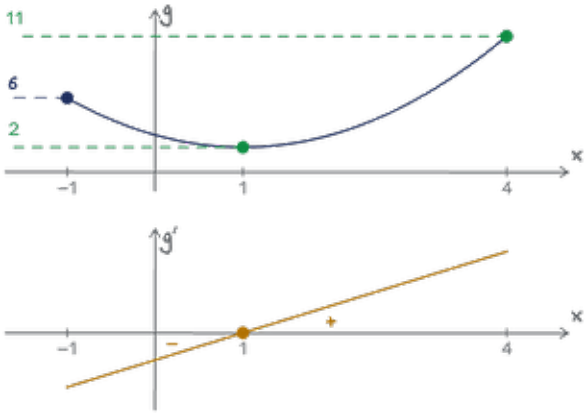
The Candidates Test AP Calculus 5.5

[a, b]: always a peak?

Max, min → compare all candidates

Justify

1. WHERE CAN A MAX HIDE?



$g(x) = x^2 - 2x + 3$ on $[-1, 4]$

valley $x = 1$ · no peak

right end: $x = 4$ highest

continuous on $[-1, 4]$ → EVT → max, min exist

Inside → turns there → critical point

Not inside → hits a wall → an endpoint

Candidates = critical points inside + both ends

Open end near 11, never reached → maybe no max

$g = x^2 - 2x + 3$, $[-1, 4]$ → compare

$g' = 2x - 2$ $g'(1) = 0$, − → +

Candidates → −1, 1, 4

$g(-1) = 1 + 2 + 3 = 6$ $g(1) = 2$

$g(4) = 16 - 8 + 3 = 11$

6, 2, 11 max 11 at 4 min 2 at 1

The Candidates Test

3. WHAT TO WRITE

1. f continuous on [a, b]
2. Critical x inside + a, b
3. f(x) for every candidate → compare
4. Value and where → max g = 11 at x = 4

f′ from + to − → beats neighbors only

Whole interval down, then up → that bottom = min

Motion → same list for x(t) and v(t)

PROBLEM 1 OF 3

Absolute max and min

Let $f(x) = 2x^3 - 9x^2 + 12x$ on $[0, 3]$.
Find the absolute maximum and minimum values of f. Justify your answer.

$$\frac{d}{dx}x^n = nx^{n-1}$$

Solution Candidates

(a)

List x → f(x) → rank

Polynomial → continuous [0, 3]

$f'(x) = 6x^2 - 18x + 12$

$= 6(x - 1)(x - 2) = 0 \rightarrow x = 1, 2$ in $[0, 3]$

4 candidates → 0, 1, 2, 3

Your turn → f(1), f(2), f(3)

Solution Values and compare

$$\frac{d}{dx}x^n = nx^{n-1}$$

(b)

$f(0) = 0$

$f(1) = 2 - 9 + 12 = 5$

$f(2) = 16 - 36 + 24 = 4$

$f(3) = 54 - 81 + 36 = 9$

0, 5, 4, 9

Absolute max 9 at $x = 3$; absolute min 0 at $x = 0$

Workbook → write every value

COMMON MISTAKE

✗ f′ from + to − at 1 → max 5

f(3) = 9 > 5 ✓ max 9 at an end

✓ Critical points + ends → compare all

PROBLEM 2 OF 3

The absolute minimum value

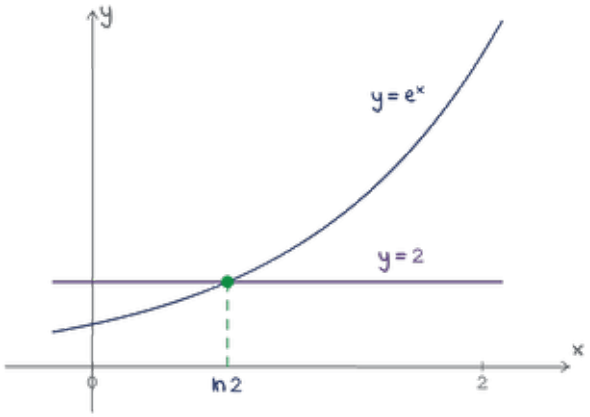
Let $f(x) = e^x - 2x$ on $[0, 2]$. What is the absolute minimum value of f?

(A) 0

(B) 1

(C) $2 - 2\ln 2$

(D) $e^2 - 4$



Solution f′ as white minus purple

(a)

Plan: sign of f′ → on all of [0, 2]

$f'(x) = e^x - 2$

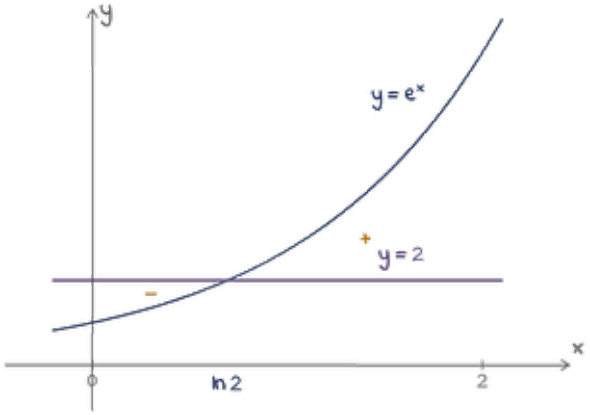
f′: white − purple

$e^x = 2 \to x = \ln 2$

$0 = \ln 1 < \ln 2 < \ln e^2 = 2$

Your turn: ln 2 · signs?

Solution Whole-interval sign



(b)

ln 2: left, white below

ln 2: right, white above

↓ [0, ln 2] ↑ [ln 2, 2] → abs. min

$f(\ln 2) = e^{\ln 2} - 2\ln 2 = 2 - 2\ln 2$

So (C)

Ends: $f(0) = 1$, $f(2) = e^2 - 4$

$e > 2.5 \rightarrow e^2 > 6.25 > 5 \rightarrow e^2 - 4 > 1$

Workbook $f' = e^x - 2$ → draw both

PROBLEM 3 OF 3

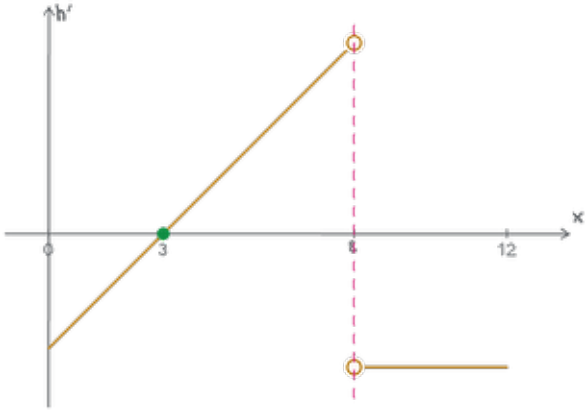
Candidates from the graph of h'

h is continuous on [0, 12]. The graph of h' is shown; h' does not exist at $x = 8$. The table gives values of h:

x: 0, 3, 8, 12

$h(x)$: 13, 4, 29, 1

Find the absolute maximum and minimum values of h on [0, 12]. Justify your answer.



Solution Candidates from the graph of h'

(a)

Graph → h′ · table → h

Plan: x → h(x) → rank

At x = 3: h′(3) = 0

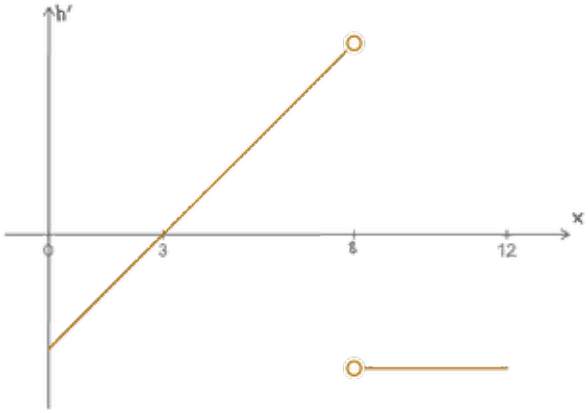
x = 8: jump, no value → h′(8) DNE

h continuous → corner → candidate 8

Ends 0, 12 all 0, 3, 8, 12

Your turn → max, min

Solution Values from the table, then compare



(b)

$h(0) = 13 \quad h(3) = 4 \quad h(8) = 29 \quad h(12) = 1$

Max 29 at $x = 8$ (where h' DNE)

Min 1 at $x = 12$ (can end)

$x = 3 \rightarrow$ relative min 4, but $4 > 1$

Workbook → candidates: f' DNE

WRAP-UP

Largest value → always a peak?

No → an end, like $g(4)$, or a corner, like $h(8)$

Sign change → neighbors only; list every candidate

Next → 5.6: which way a curve bends

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.