5.4 / Practice workbook

The First Derivative Test

Practice

Problems 1–7 / Show your reasoning

1 Let $f(x) = x^4 - 4x^3$. Find all x where f has a relative extremum, and classify each. Justify your answer.

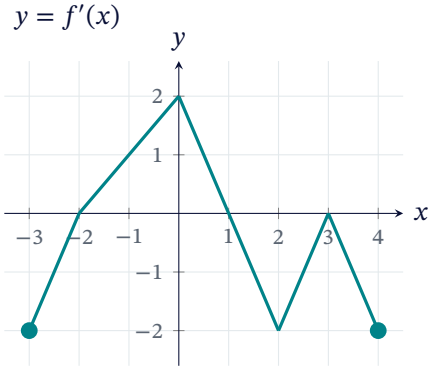
2 Let $f(x) = x + \frac{9}{x}$, $x \neq 0$. Find the relative maximum and minimum of f. Justify your answer.

3 Let $f(x) = xe^{-\frac{x^2}{2}}$. At what value of x does f have a relative maximum?

A. -1
B. 0
C. 1
D. $\sqrt{2}$

4 The graph of f', the derivative of f, is shown for $-3 \leq x \leq 4$. Find all x where f has a relative extremum, and classify each. Justify your answer.

$y = f'(x)$



5 Let $f(x) = x\sqrt{9 - x^2}$ for $-3 \leq x \leq 3$. Find the relative maximum value of f. Justify your answer.

6 Let $f(x) = x^2$ for $x \leq 1$ and $f(x) = 3 - 2x$ for $x > 1$. Does f have a relative extremum at $x = 1$? Justify your answer.

7 Let $f(x) = |x^3 - 4x|$. How many relative extrema does f have?

A. 2
B. 3
C. 4
D. 5

5.4 / Answer key

The First Derivative Test

P01 $f'(x) = 4x^3 - 12x^2 = 4x^2(x-3) = 0 \rightarrow x = 0,\ 3$
Signs of f': $-,\ -,\ +$.
f' changes from $-$ to $+$ at $x = 3$: a relative min. No sign change at $x = 0$: neither.

P02 $f'(x) = 1 - \dfrac{9}{x^2} = \dfrac{(x-3)(x+3)}{x^2}$
Signs of f': $+$ on $(-\infty, -3)$, $-$ on $(-3, 0)$ and $(0, 3)$, $+$ on $(3, \infty)$.
$+$ to $-$ at $x = -3$: a relative max. $-$ to $+$ at $x = 3$: a relative min.

P03 (C) $f'(x) = e^{-\frac{x^2}{2}} - x^2 e^{-\frac{x^2}{2}} = e^{-\frac{x^2}{2}}(1 - x^2)$
$e^{-x^2/2} > 0$; $1 - x^2$ changes from $+$ to $-$ at $x = 1$. Answer C.

P04 At $x = -2$, f' changes from $-$ to $+$: a relative min.
At $x = 1$, f' changes from $+$ to $-$: a relative max.
At $x = 3$, f' touches 0 and is negative on both sides: neither.

P05 $f'(x) = \sqrt{9 - x^2} - \dfrac{x^2}{\sqrt{9 - x^2}} = \dfrac{9 - 2x^2}{\sqrt{9 - x^2}}$
$f' = 0$ at $x = \pm\frac{3}{\sqrt{2}}$; f' changes from $+$ to $-$ at $x = \frac{3}{\sqrt{2}}$: a relative max.
$f\left(\frac{3}{\sqrt{2}}\right) = \frac{3}{\sqrt{2}} \cdot \frac{3}{\sqrt{2}} = \frac{9}{2}$

P06 f is continuous at 1: $1^2 = 1 = 3 - 2(1)$. $f'(1)$ does not exist (a corner), so $x = 1$ is a critical point.
Left of 1: $f' = 2x > 0$. Right of 1: $f' = -2 < 0$.
f' changes from $+$ to $-$ at $x = 1$, so f has a relative maximum there.

P07 (D) Draw first. Inside the bars: $x^3 - 4x = x(x-2)(x+2)$, zero at -2, 0, 2; above the axis on (-2, 0), below on (0, 2) and left of -2.
The bars flip every negative part up. At the corners x = -2, 0, 2, f' changes from - to +: three relative minimums.
The hump on (-2, 0) stays up, and the dip on (0, 2) flips into a hump: two relative maximums. $3 + 2 = 5$. Answer D.