Lecture Notes · Unit 5 · Lecture 4 AP CED 5.4

The First Derivative Test

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INTRO: IS EVERY FLAT SPOT A PEAK?

The First Derivative Test AP Calculus 5.4

f′ = 0 → a max or a min?

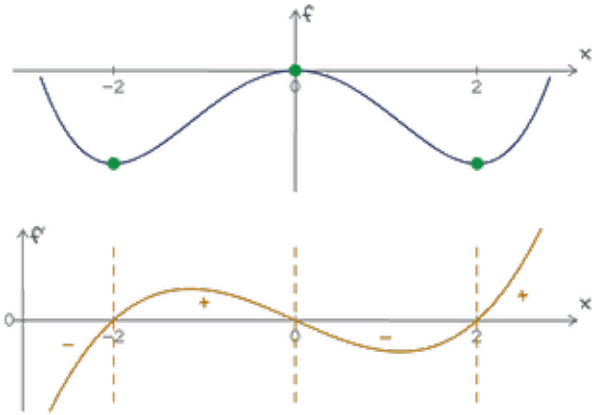
Flat tangent → always a peak or a valley?

Fake flat spots · peaks at corners

+ the one sentence to write

1. THE SIGN OF F' ON EACH SIDE

1. The sign of f ' on each side



5.2: critical point → f′ = 0 or f′ undefined

w-shaped f → flat at −2, 0, 2

f′ below → 0 at −2, 0, 2

Signs of f′ → −, +, −, +

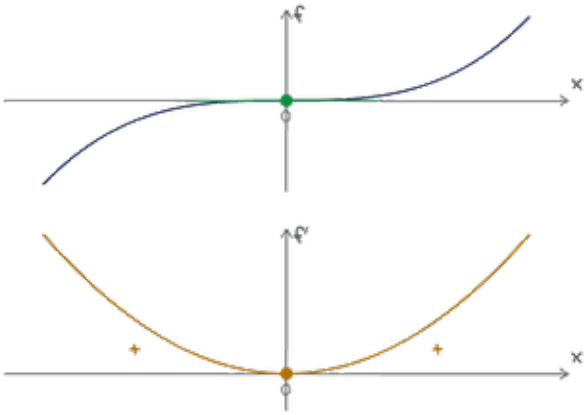
At 0 → hill, f′ from + to −

At ±2 → valleys, f′ from − to +

sign change

THE TROUBLEMAKER: Y = X^3

Flat → a peak?



Flat tangent → peak or valley? $y = x^3$

$f = x^3$ → flat at 0, keeps climbing

$f' = 3x^2$ → touches 0, stays +

No sign change → neither at 0

Critical point c, f continuous at c →

f′ from + to − → relative max

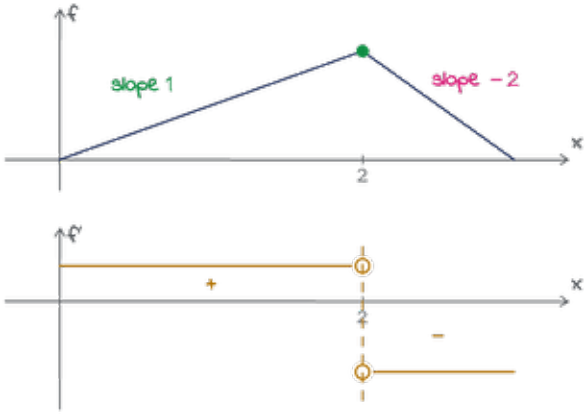
f′ from − to + → relative min

No sign change → neither

First Derivative Test (first → uses f′)

2. CORNERS: WHERE F' DOES NOT EXIST

2. A corner



Corner → f′ doesn't exist

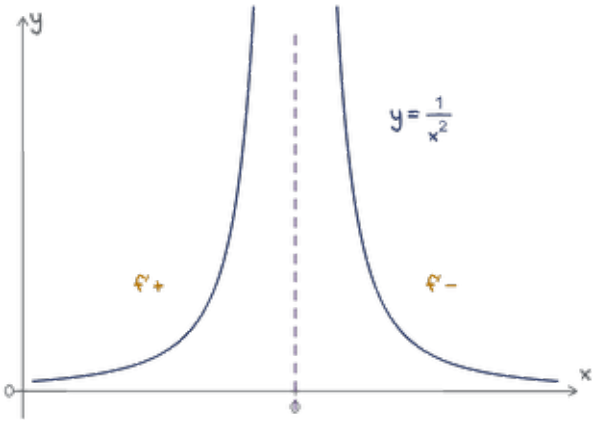
Left: slope 1

Right side → slope −2

At 2 → no single slope → f′(2) DNE

f continuous at 2, f′ from + to −

x = 2 → relative max



0: undefined → not critical → no extremum

3. THE SENTENCE TO WRITE

f′ changes from positive to negative at x = 0, so f has a relative maximum there.

f′ does not change sign at x = c, so f has neither a relative maximum nor a relative minimum there.

Name f′, not 'it' + how its sign changes

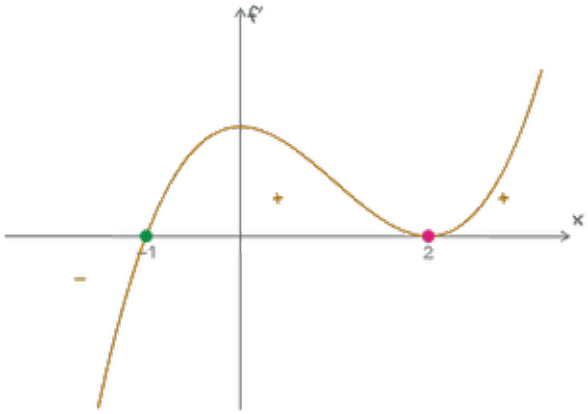
'f increases, then decreases' → shape

Reason → sign of f′

PROBLEM 1 OF 3

The sign of f′

The derivative of f is $f'(x) = (x+1)(x-2)^2$. Find all x where f has a relative extremum. Justify your answer.



Solution Find the critical points, sketch f′

(a)

Plan: f′ = 0 → sketch f′ → signs → sentence

$(x+1)(x-2)^2 = 0 \rightarrow x = -1, 2$

f′ polynomial → exists everywhere → only these two

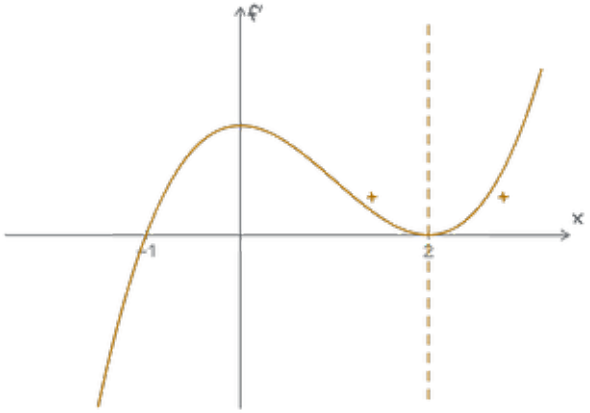
−1 → crosses · 2 → touches

Signs → −, +, +

f′ changes from negative to positive at x = −1, so f has a relative minimum there.

Your turn → the sentence for x = 2

Solution The point x = 2



(b)

Plan: sign each side of 2

Left +, right + · f'(2) = 0 → no sign change

f' does not change sign at x = 2, so f has neither a relative maximum nor a relative minimum there.

Given this graph of f' → read it the same way

Workbook → one sentence per point

COMMON MISTAKE

✗ f'(2) = 0 → an extremum at 2

✓ Check f' on both sides

✗ "f falls, then rises"

✓ f' changes sign

PROBLEM 2 OF 3

The relative maximum value

Let $f(x) = \dfrac{\ln x}{x}$. What is the relative maximum value of f?

(A) 0

(B) $\dfrac{1}{e}$

(C) 1

(D) e

$$\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$$

$$(\ln x)' = \frac{1}{x}$$

Solution Find f'

(a)

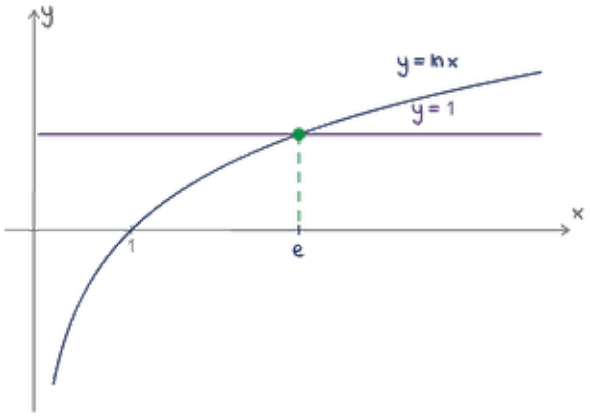
Plan: f' → sign only → value

$$f'(x) = \frac{u'v - uv'}{v^2} = \frac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2}$$

$$f'(x) = \frac{1 - \ln x}{x^2}$$

Your turn → where is $1 - \ln x = 0$?

Solution Only the sign: purple minus white



(b)

Plan: $f' = \dfrac{1 - \ln x}{x^2}$ → sign only

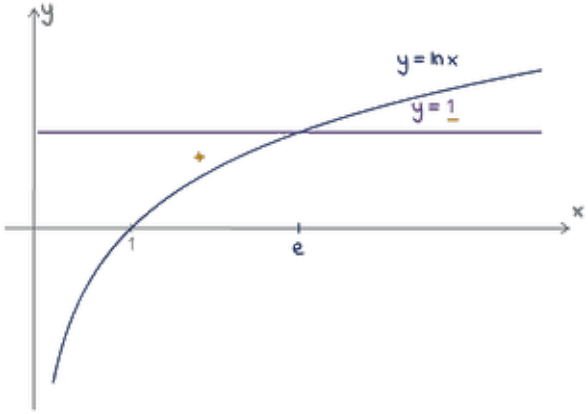
$x > 0 \rightarrow x^2 > 0 \rightarrow$ sign of $1 - \ln x$

$1 - \ln x \rightarrow y = 1$ minus $y = \ln x$

$\ln x = 1 \rightarrow x = e$

Your turn → who is on top, each side of e?

Solution Read the sign, then the value



(C)

Left of e → purple on top → +

Right of e → purple below → −

f′: + → − at e → relative max at e

$f(e) = \frac{\ln e}{e} = \frac{1}{e}$

So (B), not (D) e → that is the x

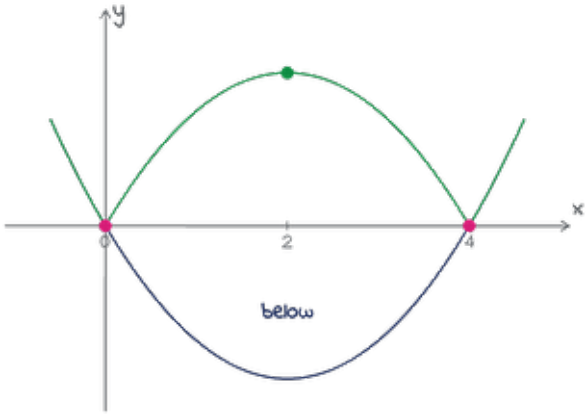
Workbook → drop what's always positive

PROBLEM 3 OF 3

How many relative extrema?

Let $f(x) = |x^2 - 4x|$. How many relative extrema does f have?

(A) 1
(B) 2
(C) 3
(D) 4



Solution Draw it, then count

Plan: draw it → count peaks and valleys

$x^2 - 4x = x(x - 4)$

Below on (0, 4) → bars flip it up

$x = 2 \rightarrow f'$ from + to − → relative max

$x = 0, 4 \rightarrow$ corners, f' DNE, − to + → relative mins

$1 + 2 = 3 \rightarrow$ (C)

Workbook → draw, flip, count

WRAP-UP

Flat → always a peak?

No → x^3: flat, but neither

Corners (f' DNE) count too

Justify → name f' + its sign change

Next → 5.5: absolute max and min on [a, b]

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.