5.4 / The First Derivative Test

Watch the lesson ➤

Using the first derivative test to determine relative (local) extrema

Learning goal

Decide whether a critical point is a relative maximum, a relative minimum, or neither from the sign change of f', corners included, and write the reason.

Key ideas

01 Flat is not always a peak: $y = x^3$ has $f'(0) = 0$, but $f'(x) = 3x^2$ is positive on both sides. No sign change: **neither**.

02 The First Derivative Test: c a critical point, f continuous at c. f' from + to −: relative **max**. f' from − to +: relative **min**. No change: neither.

03 Only the **sign** of f' matters, not its size.

04 Corners too: slope 1, then −2 at $x = 2$, so $f'(2)$ does not exist; + to − gives a relative max. Do not skip where f' does not exist.

05 Write: "f' changes from + to − at $x = 0$, so f has a relative max there." Not "f increases, then decreases."

Remember

First Derivative Test
At a critical point, a sign change of f' gives a relative max (+ to −) or min (− to +).

relative extremum
A relative maximum or a relative minimum.

Class notes

5.4 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 f’ = 0 is not the reason

✗ $f'(2) = 0$, so f has an extremum at 2. “f goes from increasing to decreasing.”

✓ No sign change of f' at 2: neither. The reason is the sign change of f'.

5.4 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
The sign of f' on each side	Critical points, the sign of f' on each side, then the sentence.	E01, E02, E03 · Practice 1, 2, 3, 5, 6, 7
From the graph of f'	Where the graph of f' crosses the axis, and which way.	Practice 4

Example 01

Video · E01

The derivative of f is $f'(x) = (x+1)(x-2)^2$.
Find all x where f has a relative extremum. Justify your answer.

> $f' = 0$ is not enough: look at the sign of f' on each side.

Example 02

Video · E02

Let $f(x) = \dfrac{\ln x}{x}$. What is the relative maximum value of f?

A. 0
B. $\dfrac{1}{e}$
C. 1
D. e

> $x^2 > 0$, so the sign of f' is the sign of its top.

Example 03

Video · E03

Let $f(x) = |x^2 - 4x|$. How many relative extrema does f have?

A. 1
B. 2
C. 3
D. 4

> Draw it: flip the part below the axis up. Count corners too.

VIDEO COMPLETE Check your example answers at the end of this booklet.

5.4 / Example answers

The First Derivative Test

E01 $f'(x) = 0$ at $x = -1$ and $x = 2$. The graph of f' crosses at -1 and touches at 2: signs $-,\ +,\ +$.
f' changes from negative to positive at $x = -1$, so f has a relative minimum there.
f' does not change sign at $x = 2$, so f has neither a relative maximum nor a relative minimum there.

E02 (B) $f'(x) = \dfrac{\frac{1}{x} \cdot x - \ln x \cdot 1}{x^2} = \dfrac{1 - \ln x}{x^2}$
$x^2 > 0$, so the sign is the sign of $1 - \ln x$: positive for $x < e$, negative for $x > e$.
f' changes from $+$ to $-$ at $x = e$: a relative max. $f(e) = \frac{\ln e}{e} = \frac{1}{e}$. Answer B.

E03 (C) $x^2 - 4x = x(x - 4) < 0$ on $(0, 4)$: that part flips up.
$x = 2$: $f' = 0$, $+$ to $-$, a relative max. $x = 0$ and $x = 4$: corners (f' does not exist), $-$ to $+$, two relative mins.
Three relative extrema. Answer C.