# 5.3 / Practice workbook

*Intervals Where a Function Is Increasing or Decreasing*

## Practice

Problems 1–7 / Show your reasoning

1 Let $f(x) = x^3 - 3x^2 - 9x + 1$. On what interval is $f$ decreasing? Justify your answer.

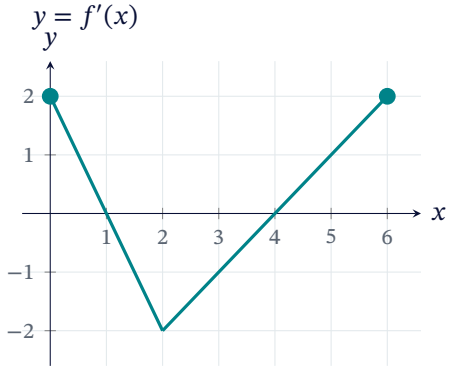
2 Let $g(x) = xe^x$. On what interval is $g$ increasing? Justify your answer.

3 Let $f(x) = x^2e^{-x}$. On which interval is $f$ increasing?

A. $(-\infty, 0)$
B. $(0, 2)$
C. $(2, \infty)$
D. $(-\infty, 2)$

4 The derivative of $f$ is $f'(x) = \dfrac{x^2 - 4}{x^2 + 1}$. On what intervals is $f$ decreasing? Justify your answer.

5 The graph of $f'$, the derivative of $f$, is shown for $0 \le x \le 6$. On what intervals is $f$ increasing? Justify your answer.




6 Let $f(x) = x\sqrt{4-x}$ for $0 \le x \le 4$. On what intervals is $f$ increasing, and on what intervals is it decreasing? Justify your answer.

7 The derivative of $f$ is $f'(x) = \sin x - \cos x$ for $0 \le x \le 2\pi$. On what interval is $f$ increasing? Justify your answer.

# 5.3 / Answer key

*Intervals Where a Function Is Increasing or Decreasing*

**P01** $f'(x) = 3x^2 - 6x - 9 = 3(x-3)(x+1)$
$f' < 0$ on $(-1, 3)$, so $f$ is decreasing on $(-1, 3)$.

**P02** $g'(x) = e^x + xe^x = e^x(x+1)$
$e^x > 0$, so $g' > 0$ on $(-1, \infty)$: g is increasing on $(-1, \infty)$.

**P03** (B) $f'(x) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2-x)$
$e^{-x} > 0$; $x(2-x) > 0$ on $(0, 2)$. Answer B.

**P04** $x^2 + 1 > 0$, so the sign of $f'$ is the sign of $x^2 - 4 = (x-2)(x+2)$.
$f' < 0$ on $(-2, 2)$, so $f$ is decreasing on $(-2, 2)$.

**P05** The graph of $f'$ is above the $x$-axis on $(0, 1)$ and $(4, 6)$.
$f' > 0$ on $(0, 1)$ and $(4, 6)$, so $f$ is increasing there.

**P06** $f'(x) = \sqrt{4-x} - \dfrac{x}{2\sqrt{4-x}} = \dfrac{8-3x}{2\sqrt{4-x}}$

The bottom is positive on $(0, 4)$; the top $8 - 3x$ is 0 at $x = \frac{8}{3}$.
$f' > 0$ on $\left(0, \frac{8}{3}\right)$: increasing. $f' < 0$ on $\left(\frac{8}{3}, 4\right)$: decreasing.

**P07** Draw $y = \sin x$ and $y = \cos x$. They meet where $\tan x = 1$: $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$.
Between them $\sin x$ is above $\cos x$, so $f' > 0$ on $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$: $f$ is increasing there.