Lecture Notes · Unit 5 · Lecture 3 AP CED 5.3

Increasing and decreasing

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: BETWEEN THE PINS

Increasing and Decreasing · AP Calculus 5.3

f up or down? → sign of f'

Pin: f' = 0 or DNE

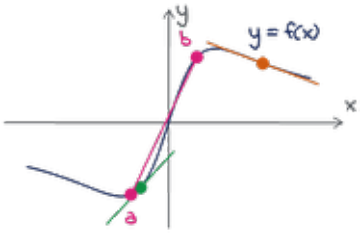
Sign → from the graph of f'

Reason → one sentence

IDEA 1: THE SIGN OF F'

Uphill, and the slope

Walk left to right: uphill?



Tilts up → slope > 0 → f' > 0

Tilts down → slope < 0 → f' < 0

f' > 0 on an interval → f increasing

f' < 0 → f decreasing

Unit 4: one instant → now: a whole interval

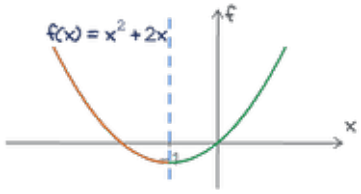
Secant → MVT: $f(b) - f(a) = f'(c)(b - a)$

$f'(c) > 0$, $b - a > 0$ → $f(b) > f(a)$

IDEA 2: SPLIT, THEN READ THE GRAPH OF F'

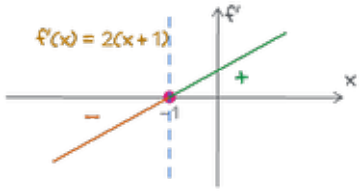
f on top, f' below

$f(x) = x^2 + 2x$



$f'(x) = 2x + 2 = 2(x + 1)$

$f' = 0 \to x = -1$ → critical point



Slope of f = height of f'

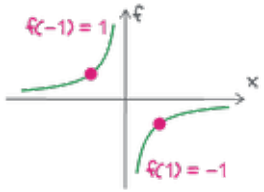
f' below → f decreasing

f' above → f increasing

Only at f' = 0, DNE, or a break

None → one sign → f one way

Break: $f = -\frac{1}{x}$, $f' = \frac{1}{x^2} > 0$



$(-\infty, 0)$, $(0, \infty)$ → increasing separately

Decreasing $(-\infty, -1)$ → − piece

Increasing $(-1, \infty)$ → + piece

IDEA 3: THE SENTENCE

The reason is a sentence

f′ picture → tool, not the reason

$f(x) = x^2 + 2x \rightarrow$

"$f'(x) > 0$ on $(-1, \infty)$,

so f is increasing."

Name f′, not "it" → slope of f

Name the interval

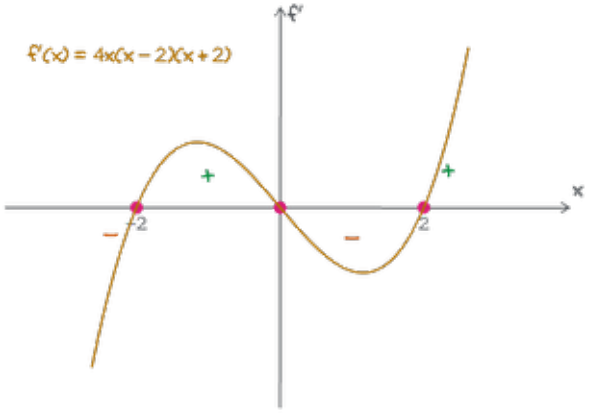
"Up, so increasing" → not a reason

PROBLEM 1 OF 3

Where is f increasing?

Let $f(x) = x^4 - 8x^2$.

On what intervals is f increasing? Justify your answer.



Solution Draw f′, read its signs, then the sentence

Plan: f′ → draw → signs → sentence

$f'(x) = 4x^3 - 16x$

$= 4x(x^2 - 4) = 4x(x-2)(x+2)$

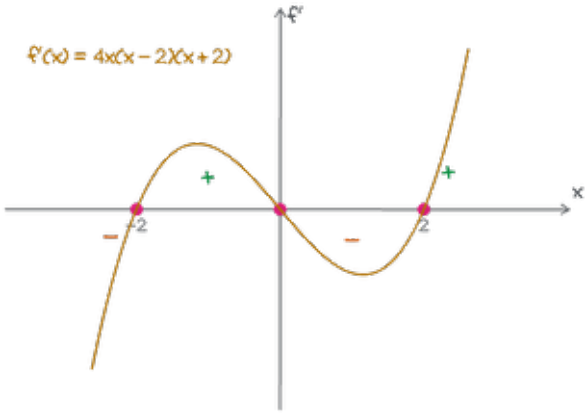
$f' = 0 \rightarrow x = -2,\ 0,\ 2$

Cubic, leading term + → up on the right

Test values → into f′, not f

Your turn: the sentence.

Solution Draw f′, read its signs, then the sentence

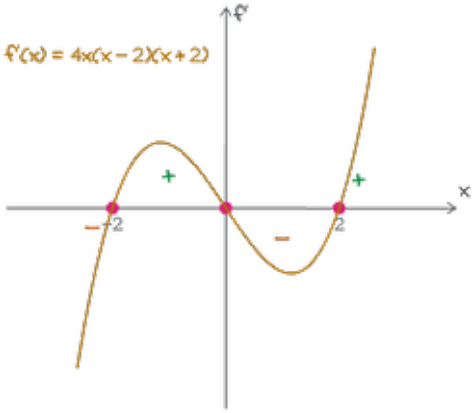


Plan: + pieces → sentence

$f'(x) > 0$ on $(-2, 0)$, $(2, \infty)$,

so f is increasing there.

Careful the sentence



✗ Common mistake

2 intervals only

No sentence → no reason

✓ Do this

$f'(x) > 0$ on $(-2, 0)$, $(2, \infty)$,

so f increases.

Workbook: f′ → sentence

PROBLEM 2 OF 3

Read the graph of f'

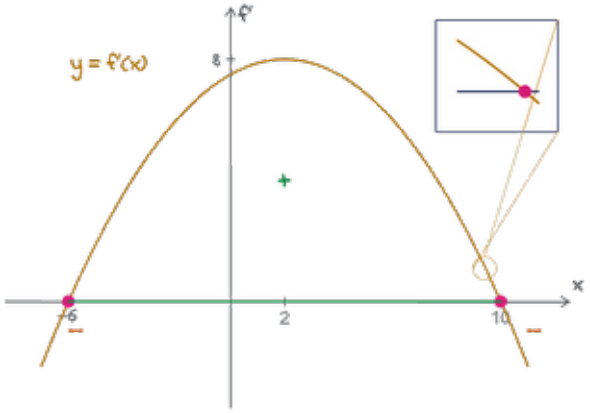
The graph of f', the derivative of f, is shown for $-7 \le x \le 11$. On which interval is f increasing?

(A) $(-7, 2)$

(B) $(-6, 10)$

(C) $(2, 11)$

(D) $(10, 11)$



Solution f increasing where f' > 0: above the x-axis

It's f′ → signs first

Plan: where f′ > 0

f′ > 0 → above the x-axis → (−6, 10)

Still above → f still rising

(A): f′ going up ≠ f going up

(C) (2, 11) → past 10, f′ below

(D) (10, 11) → f′ < 0, decreasing

So (B), (−6, 10).

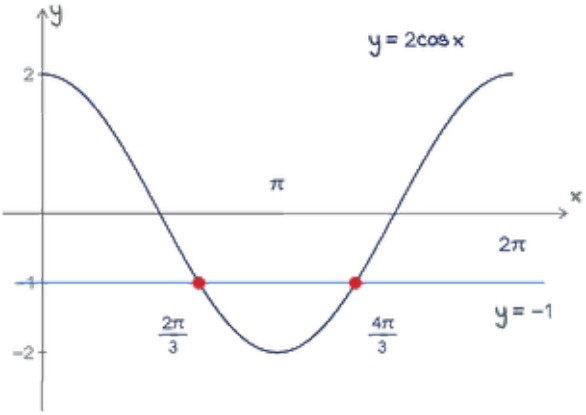
Above axis, not the rise

PROBLEM 3 OF 3

Where is f decreasing?

Let $f(x) = x + 2\sin x$ for $0 \le x \le 2\pi$.

On what interval is f decreasing? Justify your answer.



Solution

f' = 2cos x - (-1): the curve against the line, then the sentence

Plan: curve − line → signs

$f' = 1 + 2\cos x$

$= 2\cos x - (-1)$

Curve − flat line → which is on top?

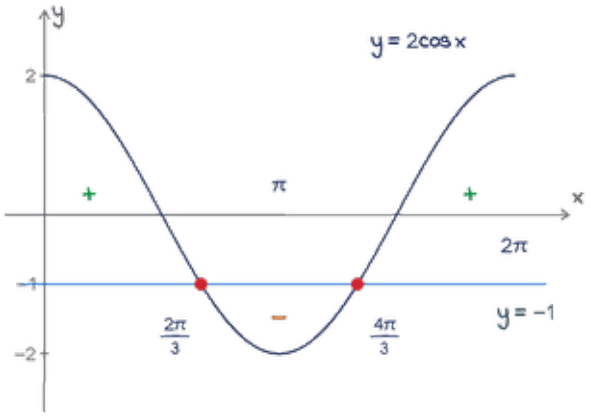
Meet: $2\cos x = -1 \to \cos x = -\frac{1}{2}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Your turn: read the signs.

Solution

f' = 2cos x - (-1): the curve against the line, then the sentence



Plan: $f'(x) = 2\cos x - (-1)$ → signs

white above → + ; white below → − ; then +

$f'(x) < 0$ on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$

so f is decreasing there.

workbook → draw both graphs

→ $\sin x - \cos x$

WRAP-UP

Between the pins: up or down?

f' above the axis → up; below → down

The reason: name f', name the interval

workbook first → 5.4 → does f turn?

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.