5.3 / Intervals Where a Function Is Increasing or Decreasing

Watch the lesson ➤

Determining intervals on which a function is increasing or decreasing

Learning goal

Find where f is increasing or decreasing from the sign of f', and write the sentence that earns the point.

Key ideas

01 Uphill: the tangent tilts up, slope > 0. If $f' > 0$ on an interval, f is **increasing** there; if $f' < 0$, **decreasing**.

02 Split where $f' = 0$ or f' does not exist: the sign of f' can switch only there. $f(x) = x^2 + 2x$: $f'(x) = 2(x + 1)$, decreasing on $(-\infty, -1)$, increasing on $(-1, \infty)$.

03 Up or down depends only on the **sign** of f', not on whether f' itself goes up.

04 The picture of f' is a tool, not the reason. Write: "$f' > 0$ on $(-1, \infty)$, so f is increasing there." Name f' (not "it") and name the interval.

Remember

increasing
On an interval where $f' > 0$, f goes up from left to right.

decreasing
On an interval where $f' < 0$, f goes down from left to right.

Class notes

5.3 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 A sign chart with no sentence

✗ A sign chart, then: increasing on $(-2, 0)$ and $(2, \infty)$.

✓ "$f' > 0$ on $(-2, 0)$ and $(2, \infty)$, so f is increasing there."

5.3 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Split, test, write the sentence	Zeros of f' (and where it does not exist), the sign on each piece, then the sentence.	E01, E03 · Practice 1, 2, 3, 4, 6, 7
From the graph of f’	f increases where the graph of f' is above the axis.	E02 · Practice 5

Example 01

Video · E01

Let $f(x) = x^4 - 8x^2$.
On what intervals is f increasing? Justify your answer.

> Factor f', mark where it is 0, read the signs, then write the sentence.

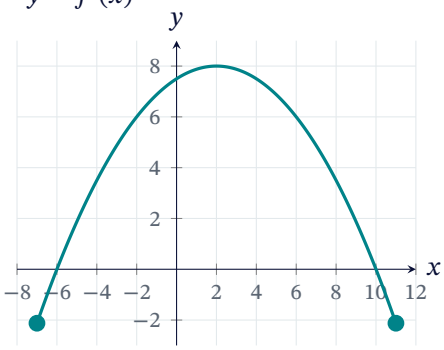
Example 02

Video · E02

The graph of f', the derivative of f, is shown for $-7 \le x \le 11$. On which interval is f increasing?

f goes up where f' is above the axis, not where f' goes up.

$y = f'(x)$



A. $(-7, 2)$
B. $(-6, 10)$
C. $(2, 11)$
D. $(10, 11)$

Example 03

Video · E03

Let $f(x) = x + 2\sin x$ for $0 \le x \le 2\pi$.
On what interval is f decreasing? Justify your answer.

$f'(x) = 2\cos x - (-1)$: where is the curve $y = 2\cos x$ below the line $y = -1$?

VIDEO COMPLETE Check your example answers at the end of this booklet.

5.3 / Example answers

Intervals Where a Function Is Increasing or Decreasing

E01 $f'(x) = 4x^3 - 16x = 4x(x-2)(x+2) = 0 \rightarrow x = -2,\ 0,\ 2$
The graph of f' is a cubic, up on the right: signs $-,\ +,\ -,\ +$.
$f' > 0$ on $(-2, 0)$ and $(2, \infty)$, so f is increasing there.

E02 (B) f is increasing where $f' > 0$: the graph of f' is above the x-axis on $(-6, 10)$.
f' going up is not f going up: past 10, $f' < 0$. Answer B.

E03 $f'(x) = 1 + 2\cos x = 2\cos x - (-1)$
$2\cos x = -1$ at $x = \frac{2\pi}{3}$ and $x = \frac{4\pi}{3}$; between them the curve is below the line.
$f' < 0$ on $\left(\frac{2\pi}{3}, \frac{4\pi}{3}\right)$, so f is decreasing there.