5.2 / Practice workbook

Extreme Value Theorem and Critical Points

Practice

Problems 1–7 / Show your reasoning

1 Find all critical points of $f(x) = 2x^3 + 3x^2 - 36x$.

2 Find the critical point of $g(x) = xe^{-x}$.

3 How many critical points does $f(x) = x^{\frac{1}{3}}(x + 4)$ have?

A. 0
B. 1
C. 2
D. 3

4 Which function must have an absolute maximum value on the given interval?

A. $\frac{1}{x}$ on $[-1, 1]$
B. $\tan x$ on $\left[0, \frac{\pi}{2}\right)$
C. x^2 on $(0, 2)$
D. $\sqrt{x+2}$ on $[-2, 2]$

5 Let $f(x) = |x^2 - 4|$ on $[-3, 3]$. Find all critical points of f, and say for each whether $f' = 0$ or f' does not exist.

6 Find the critical points of $f(x) = x - 2\cos x$ on $[0, 2\pi]$.

7 The function g is differentiable on $[0, 8]$. Selected values of g are in the table.

x	0	2	4	6	8
$g(x)$	9	3	6	8	4

(a) Must g have an absolute maximum value and an absolute minimum value on $[0, 8]$? Justify your answer.

(b) Must the absolute minimum occur at a critical point in $(0, 8)$? Justify your answer.

(c) Must the absolute maximum occur at a critical point in $(0, 8)$? Explain.

5.2 / Answer key

Extreme Value Theorem and Critical Points

P01 $f'(x) = 6x^2 + 6x - 36 = 6(x+3)(x-2)$
$f'(x) = 0$ at $x = -3$ and $x = 2$; f' exists everywhere. Critical points: $x = -3,\ 2$.

P02 $g'(x) = e^{-x} - xe^{-x} = e^{-x}(1-x)$
$e^{-x} > 0$, so $g'(x) = 0$ only at $x = 1$. Critical point: $x = 1$.

P03 (C) $f'(x) = \dfrac{4(x+1)}{3x^{\frac{2}{3}}}$
$x = -1$ ($f' = 0$) and $x = 0$ (f' does not exist, $f(0) = 0$). Two critical points: C.

P04 (D) (A) breaks at 0. (B) and (C) are not closed intervals: no promise.
(D) $\sqrt{x+2}$ is continuous on the closed interval $[-2, 2]$. By the EVT, D.

P05 On $(-2, 2)$, $f(x) = 4 - x^2$ (flipped up), so $f'(x) = -2x = 0$ at $x = 0$.
At $x = -2$ and $x = 2$ the graph has corners: f' does not exist.
Critical points: $x = 0$ ($f' = 0$); $x = -2,\ 2$ (f' does not exist).

P06 $f'(x) = 1 + 2\sin x = 0 \ \rightarrow\ \sin x = -\dfrac{1}{2}$
On $[0, 2\pi]$: $x = \dfrac{7\pi}{6}$ and $x = \dfrac{11\pi}{6}$.

P07 (a) g is differentiable, so continuous on [0, 8]. By the EVT it has an absolute maximum and an absolute minimum there. Yes.
(b) The minimum is at most g(2) = 3, below both ends g(8) = 4 and g(0) = 9. So it is inside: a relative minimum where g'(c) = 0, a critical point. Yes.
(c) The maximum is at least 9 = g(0), an end value, and no table value is larger. The maximum may be g(0) itself, at an end. Not necessarily.