Lecture Notes · Unit 5 · Lecture 2 AP CED 5.2

EVT, relative vs absolute, critical points

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INTRO: A HOTTEST MOMENT?

The EVT and Critical Points AP Calculus 5.2

Today: highest, lowest value

6 a.m. → 6 p.m. → a hottest moment?

Of course? → 2 hidden conditions

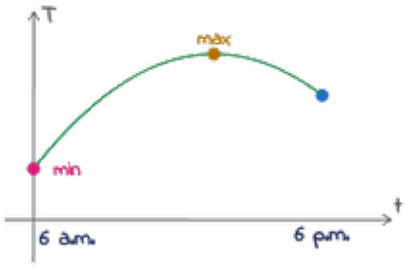
When it must exist, where to look

IDEA 1: THE EXTREME VALUE THEOREM

When must there be a top?

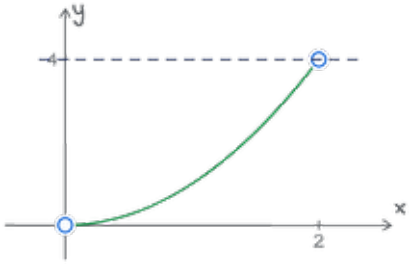
1. A top and a bottom: when for sure?

6 a.m. → 6 p.m.



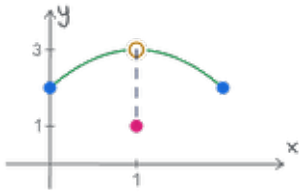
No jumps, both ends → top, bottom

Open ends → x^2 on $(0, 2)$



Never 4 → no max, no min

→ need the closed interval $[a, b]$



→ need f cont. on $[a, b]$, no holes, no jumps

If f is continuous on $[a, b]$,

→ at least one max

and at least one min

→ Extreme Value Theorem

A tie is fine

A max exists

Does not say where

IVT VS MVT VS EVT

Three theorems, one promise: 'there is'

theorem	IVT	MVT	EVT
there is	a value	a slope	a max, a min

Three theorems that sound alike

All f cont. on $[a, b]$ (MVT: also diff.)

All three: 'there is one' → never where

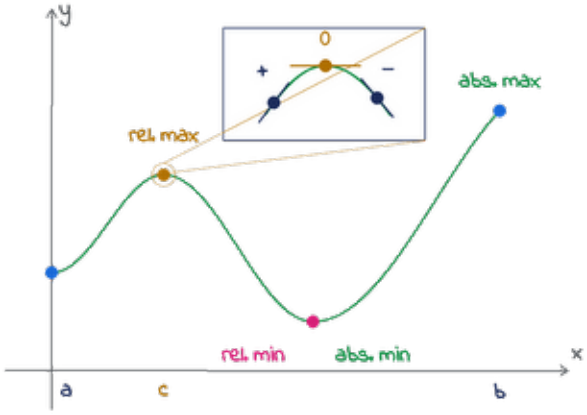
→ two kinds of tops

IDEA 2: RELATIVE VS ABSOLUTE

Tops: nearby, of all, and inside

2. Highest nearby or highest of all?

nearby vs all



Hill → highest nearby

→ a relative max (local max)

valley → a relative min

Right end → absolute max

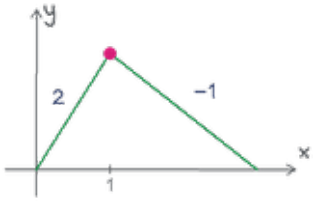
valley → also absolute min

An end can be abs. max or min

3. A top inside: what does it look like?

inside, up close

Smooth top: $f'(c) = 0$



Sharp: 2 → −1 → $f'(c)$ DNE

Top inside: $f' = 0$ or f' DNE

c in the domain of f, not an end, $f'(c) = 0$ or DNE

→ c is a critical point of f

Ends: not on the list → check them separately

Rel. max or min inside → critical point

Not always → a place to look (Ex. 3)

Find: $f'(x) = 0$ and f' DNE

PROBLEM 1 OF 3

Find all critical points

Let $f(x) = x^{\frac{2}{3}}(x-5)$.

Find all critical points of f.

Power rule

$\frac{d}{dx}x^n = nx^{n-1}$

$x^a \cdot x^b = x^{a+b}$

$x^{-a} = \frac{1}{x^a}$

Solution Write f' as one fraction: top = 0, bottom = 0

Plan: f' as one fraction → top = 0, bottom = 0

$$f(x) = x^{\frac{2}{3}} \cdot x - 5x^{\frac{2}{3}}$$

$$= x^{\frac{2}{3}+\frac{3}{3}} - 5x^{\frac{2}{3}} = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$$

$$f'(x) = \frac{5}{3}x^{\frac{5}{3}-1} - 5 \cdot \frac{2}{3}x^{\frac{2}{3}-1}$$

$$= \frac{5}{3}x^{\frac{2}{3}} - \frac{10}{3}x^{-\frac{1}{3}}$$

$$= \frac{5}{3}x^{\frac{2}{3}} - \frac{10}{3x^{\frac{1}{3}}}$$

Solution Write f' as one fraction: top = 0, bottom = 0

So far

$f'(x) = \frac{5}{3}x^{\frac{2}{3}} - \frac{10}{3x^{\frac{1}{3}}}$

$x^a \cdot x^b = x^{a+b}$

Plan: bottom $3x^{\frac{1}{3}}$

$$\frac{5x^{\frac{2}{3}}}{3} \cdot \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}}$$

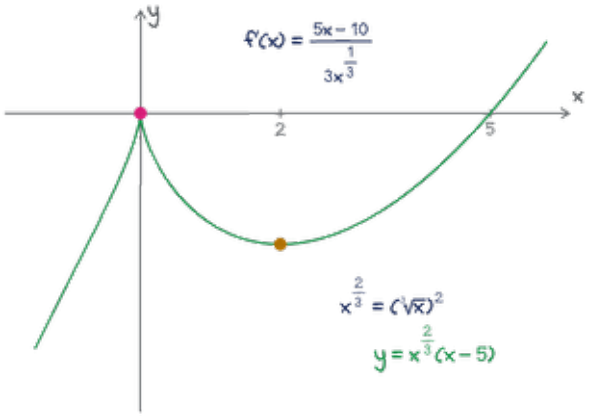
$$x^{\frac{2}{3}} \cdot x^{\frac{1}{3}} = x^{\frac{3}{3}} = x$$

$$f'(x) = \frac{5x}{3x^{\frac{1}{3}}} - \frac{10}{3x^{\frac{1}{3}}}$$

$$= \frac{5x-10}{3x^{\frac{1}{3}}}$$

Your turn: top = 0, bottom = 0

Solution Top = 0, bottom = 0, then check the domain



$5x - 10 = 0 \rightarrow x = 2 \rightarrow f'(2) = 0$

$3x^{\frac{1}{3}} = 0 \rightarrow x = 0 \rightarrow f'(0)$ DNE

$f(0) = 0^{\frac{2}{3}}(0-5) = 0 \rightarrow 0$ in the domain

No $f(0) \rightarrow$ not critical

Critical pts: $x = 0, x = 2$

Sharp at 0, flat at 2

Workbook: both kinds, every time

PROBLEM 2 OF 3

Inside or at an end?

h is differentiable.

(a) Must h have an absolute max and min on $[0, 10]$?

(b) Must each occur at a critical point in $(0, 10)$? Justify your answer.

x	0	2	6	10
$h(x)$	5	9	1	6

Solution (a) The theorem's condition, then the EVT

(a)

Plan: cont. on $[0, 10]$? → EVT

h differentiable → continuous on $[0, 10]$

EVT → at least one abs. max and abs. min: Yes

Your turn: max at an end?

Solution

(b) Compare with the ends: inside, or at an end?

x	0	2	6	10
$h(x)$	5	9	1	6

(b)

Plan: vs. the two ends

$h(0) = 5, h(10) = 6$

$h(2) = 9 \rightarrow$ the max is 9 or more

Max $\geq h(2) = 9 > 5, 6 \rightarrow$ not at an end

→ inside → rel. max → critical point, $h' = 0$

Min $\leq h(6) = 1 < 5, 6 \rightarrow$ inside too

Both at critical points: Yes

Workbook: the ends first

PROBLEM 3 OF 3

Which must be true?

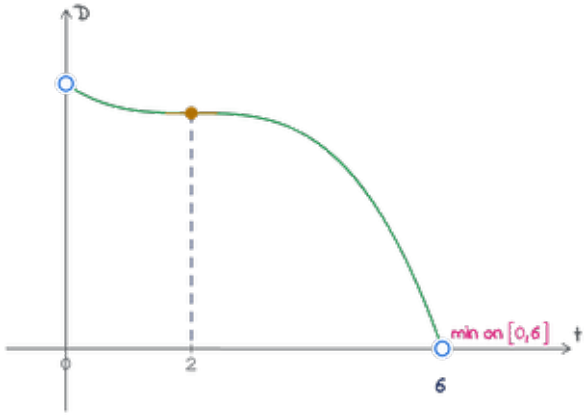
$D(t)$ is the depth of water in a tank, in feet, for $0 \le t \le 6$ hours. D is differentiable. Which of the following must be true?

(A) The absolute minimum of D on $[0, 6]$ is at a critical point.

(B) If $D'(2) = 0$, then D has a relative max or min at $t = 2$.

(C) D has an absolute minimum on $[0, 6]$.

(D) D has an absolute minimum on $(0, 6)$.



Solution

A theorem for 'must'; one picture for 'not always'

Plan: must → a theorem; out → one picture

(C) D diff. → cont. [0,6] → EVT → must

one tank: it only drains

(A) min at t = 6, an end → out

(B) D'(2) = 0

→ neither a rel. max nor a rel. min → out

(D) on (0,6) → never 0 → no min

open interval → no guarantee → out

So (C)

Workbook: closed or open? Say why

write the reason

✗ Common mistake

D cont. → has a min

✓ Do this

D diff, so cont. on [0,6].

By the EVT, D has a min on [0,6].

WRAP-UP

A hottest moment? Yes

Yes: continuous on a closed interval → EVT → at least one

At least one → where? The theorem doesn't say

Inside → critical points: f' = 0 or DNE + the ends

'Must' → a theorem behind it

Next → 5.3: up, down, the sign of f'

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.