5.2 / Extreme Value Theorem and Critical Points

Watch the lesson ➤

The Extreme Value Theorem, global versus local extrema, and critical points

Learning goal

Say why an absolute maximum and minimum must exist (the Extreme Value Theorem), tell relative from absolute, and find the critical points where they can be.

Key ideas

01 The Extreme Value Theorem: if f is continuous on $[a, b]$, then f has at least one **maximum** value and at least one **minimum** value on $[a, b]$. It says a max exists, not where.

02 Both conditions matter: x^2 on $(0, 2)$ gets close to 4 but never reaches it (not closed); a hole at the top leaves no top (not continuous).

03 A hill is a **relative** (local) max: highest nearby. The highest of all is the **absolute** (global) max, and it can be at an endpoint.

04 A **critical point**: c in the domain with $f'(c) = 0$ (flat tangent) or $f'(c)$ does not exist (a corner). A relative max or min is at a critical point; not every critical point is one.

Remember

Extreme Value Theorem
f continuous on $[a, b]$: at least one max value and one min value on $[a, b]$.

critical point
c in the domain of f with $f'(c) = 0$ or $f'(c)$ undefined.

absolute vs relative
Highest (lowest) of all on the interval vs highest (lowest) nearby.

Class notes

5.2 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 A reason with no theorem

✗ D is continuous, so it has a minimum.

✓ D is differentiable, so it is continuous on $[0, 6]$. By the EVT, D has at least one minimum value there.

5.2 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Find the critical points	Write f' as one piece: where it is 0, and where it does not exist (with c in the domain).	E01 · Practice 1, 2, 3, 5, 6
Does the EVT apply?	Continuous, and a closed interval: both, or no promise.	E02, E03 · Practice 4, 7

Example 01

Video · E01

Let $f(x) = x^{\frac{2}{3}}(x-5)$.
Find all critical points of f.

> Write f' as one fraction. The top gives $f' = 0$; the bottom gives f' DNE.

Example 02

Video · E02

h is differentiable.
(a) Must h have an abs. max and min on $[0, 10]$?
(b) Each at a critical point in $(0, 10)$? Justify.

x	0	2	6	10
$h(x)$	5	9	1	6

> The EVT says a max exists. The table says whether it can be at an end.

Example 03

Video · E03

Water depth $D(t)$ ft, $0 \le t \le 6$ hr. D is differentiable. Which must be true?

A. The absolute minimum of D on $[0, 6]$ is at a critical point.
B. If $D'(2) = 0$, then D has a relative max or min at $t = 2$.
C. D has an absolute minimum on $[0, 6]$.
D. D has an absolute minimum on $(0, 6)$.

> Plan: must → a theorem; not must → one picture

VIDEO COMPLETE Check your example answers at the end of this booklet.

5.2 / Example answers

Extreme Value Theorem and Critical Points

E01 $f(x) = x^{\frac{5}{3}} - 5x^{\frac{2}{3}}$, $\quad f'(x) = \frac{5}{3}x^{\frac{2}{3}} - \frac{10}{3}x^{-\frac{1}{3}} = \frac{5x-10}{3x^{\frac{1}{3}}}$

Top $= 0$: $5x - 10 = 0$, $x = 2$. Bottom $= 0$: $x = 0$, so $f'(0)$ does not exist.

$f(0) = 0$, so 0 is in the domain. Critical points: $x = 0$ and $x = 2$.

E02 (a) h is differentiable, so it is continuous on $[0, 10]$. By the EVT, yes.

(b) The max is at least $h(2) = 9$, more than $h(0) = 5$ and $h(10) = 6$: not at an end.

The min is at most $h(6) = 1$, less than both ends: inside too. Inside, an extremum is at a critical point. Yes.

E03 (C) D differentiable $\rightarrow$ continuous on $[0, 6]$; EVT $\rightarrow$ an absolute min on $[0, 6]$: must.

One tank that only drains: the lowest point is at $t = 6$, an end, not a critical point, so (A) no.

(B) $D'(2) = 0$, but neither a relative max nor min: no. (D) On $(0, 6)$ it gets close to the lowest value, never reaches it: no.

So (C). Closed + continuous gives the promise.