5.1 / Practice workbook

Using the Mean Value Theorem

Practice

Problems 1–7 / Show your reasoning

1 Let $f(x) = x^2 - 4x + 1$ on $[0, 3]$. Find the value of c in $(0, 3)$ that satisfies the conclusion of the Mean Value Theorem.

2 Let $g(x) = x^2 - 6x + 5$ on $[1, 5]$. Explain why Rolle's Theorem applies, and find the value of c it guarantees.

3 Let $f(x) = \sqrt{x + 1}$. What value of c satisfies the conclusion of the Mean Value Theorem on $[0, 3]$?

A. $\frac{1}{2}$
B. 1
C. $\frac{5}{4}$
D. $\frac{3}{2}$

4 The function h is differentiable. Selected values of $h(t)$ are shown in the table.

t	0	2	5	8
$h(t)$	10	16	16	7

(a) Must there be a value of t, $2 < t < 5$, with $h'(t) = 0$? Justify your answer.

(b) Must there be a value of t, $5 < t < 8$, with $h'(t) = -3$? Justify your answer.

(c) Must there be a value of t, $5 < t < 8$, with $h(t) = 12$? Justify your answer.

5 Let $f(x) = |x - 1|$ on $[0, 3]$. (a) Show that no c in $(0, 3)$ has $f'(c)$ equal to the average rate of change of f on $[0, 3]$. (b) Explain why this does not contradict the Mean Value Theorem.

6 Let $f(x) = x^2$ for $x \le 1$ and $f(x) = 2x - 1$ for $x > 1$. (a) Show that the Mean Value Theorem applies to f on $[0, 2]$. (b) Find the value of c it guarantees.

7 Which function satisfies the hypotheses of the Mean Value Theorem on the given interval?

A. $f(x) = |x + 1|$ on $[-3, 1]$

B. $g(x) = \sqrt{4 - x}$ on $[0, 4]$

C. $h(x) = x^{\frac{1}{3}}$ on $[-1, 8]$

D. $k(x) = \dfrac{2}{x - 3}$ on $[1, 4]$

5.1 / Answer key

Using the Mean Value Theorem

P01 polynomial -> continuous on [0, 3], differentiable on (0, 3)

$$\frac{f(3)-f(0)}{3-0}=\frac{-2-1}{3}=-1,\quad 2c-4=-1,\quad c=\frac{3}{2}$$

P02 polynomial -> continuous and differentiable; g(1) = g(5) = 0

$$g'(c)=2c-6=0,\quad c=3$$

P03 (C) $\frac{f(3)-f(0)}{3-0}=\frac{2-1}{3}=\frac{1}{3}$

$$\frac{1}{2\sqrt{c+1}}=\frac{1}{3}$$

$$c=\frac{5}{4}$$

P04 h is differentiable, so it is continuous on every interval.

(a) $h(2)=h(5)=16$. By Rolle's Theorem, yes.

(b) $\frac{h(8)-h(5)}{8-5}=\frac{7-16}{3}=-3$. By the MVT, yes.

(c) $7<12<16$ between $t=5$ and $t=8$. By the IVT, yes.

P05 $\frac{f(3)-f(0)}{3-0}=\frac{2-1}{3}=\frac{1}{3}$

$f'(x)=-1$ for $x<1$ and $f'(x)=1$ for $x>1$: never $\frac{1}{3}$.

f is not differentiable at $x=1$, inside $(0,3)$. The MVT does not apply, so there is no contradiction.

P06 at 1: heights 1 = 1, slopes 2 = 2 -> continuous and differentiable

$$\frac{f(2)-f(0)}{2-0}=\frac{3-0}{2}=\frac{3}{2},\quad 2c=\frac{3}{2},\quad c=\frac{3}{4}$$

P07 (B) Hypotheses: continuous on the closed interval, differentiable on the open interval.

(A) |x + 1| has a corner at x = -1, inside (-3, 1). (C) $h'(x)=1/(3x^{2/3})$ does not exist at 0, inside (-1, 8). (D) 2/(x - 3) breaks at 3, inside [1, 4].

(B) sqrt(4 - x) is continuous on [0, 4]; g'(x) = -1/(2 sqrt(4 - x)) fails only at x = 4, an endpoint, which (0, 4) leaves out. Answer B.