Lecture Notes · Unit 5 · Lecture 1 AP CED 5.1

Mean Value Theorem and Rolle's Theorem

Tap a blue play button next to a heading to open that part of the video. **▶ Watch the video**

INTRO: EXACTLY 50?

Rolle's Theorem and the MVT AP Calculus 5.1

150 mi in 3 hr → exactly 50?

20? 70? coffee?

Gut: yes → today a proof

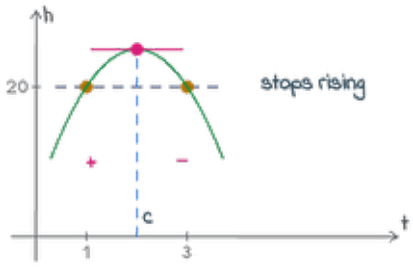
Rolle, MVT ≠ IVT → the reason

IDEA 1: ROLLE'S THEOREM

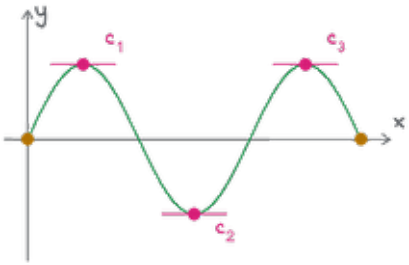
Same height at both ends

Easier: the same height

h(1) = h(3) = 20



Top c → h'(c) = 0

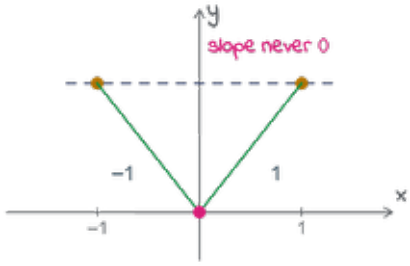


→ at least one c

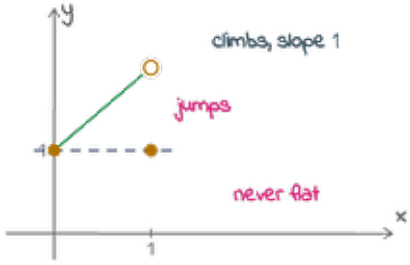
f(a) = f(b) → top or bottom c → f'(c) = 0

Same height → always flat?

|x| on [−1, 1] → a corner



→ need differentiable on (a, b)



→ need continuous on [a, b]

(i) cont. on [a, b]

(ii) diff. on (a, b)

f(a) = f(b) (3)

→ at least one c in (a, b), f'(c) = 0

→ Rolle's Theorem

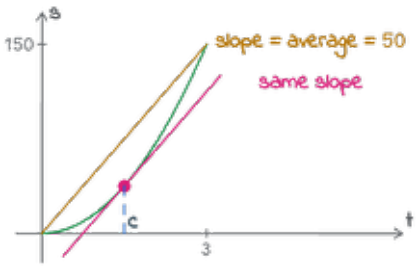
IDEA 2: TILT IT, THE MEAN VALUE THEOREM

Tilt it

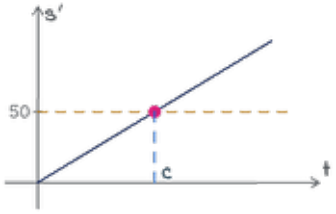
Tilt it: different heights

f(a) ≠ f(b)

Position s → average = $\frac{150-0}{3-0} = 50$



Tangent, slope 50 → s′(c) = velocity = 50



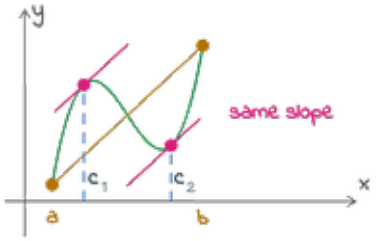
On s → 50 = slope

On s′ → same 50 = height

Slope of s′ = s″ → not in it

Secant slope = $\frac{f(b)-f(a)}{b-a}$

Touches at some c → at least one



(i) cont. on [a, b],

(ii) diff. on (a, b)

→ at least one c in (a, b),

$f'(c) = \frac{f(b)-f(a)}{b-a}$

Mean Value Theorem

Rolle → the avg. = 0 case

IDEA 3: WHAT TO WRITE

what to write

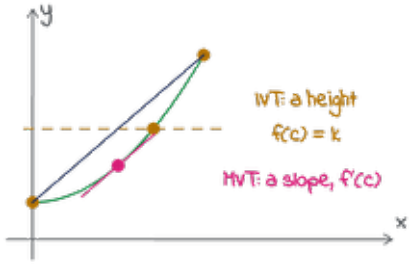
what to write

Slope everywhere → no breaks → cont.

"f is differentiable,

so it's continuous on [a, b]

and differentiable on (a, b)."



Equal ends, f′ = 0 → Rolle, too (both work)

A condition fails → no promise

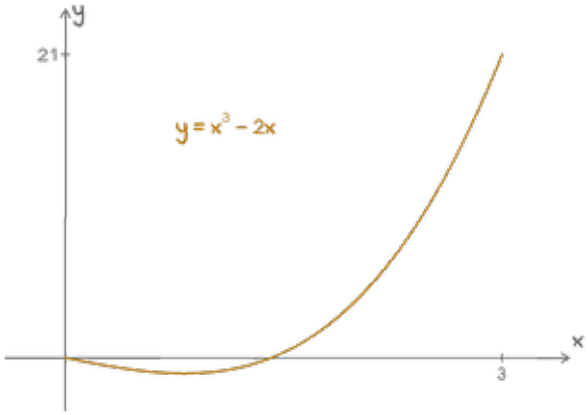
PROBLEM 1 OF 3

Find c

Let $f(x) = x^3 - 2x$ on the interval $[0, 3]$.

(a) Show f is continuous on $[0, 3]$ and differentiable on $(0, 3)$.

(b) Find all c in $(0, 3)$ that satisfy the conclusion of the MVT.



Solution (a) The conditions

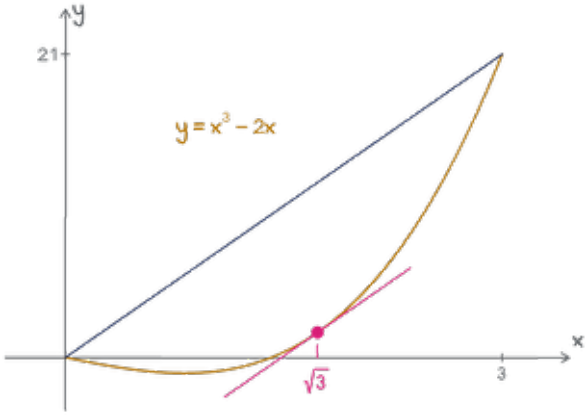
(a)

Plan: conditions → avg. → solve f′(c)

Polynomial → cont. on [0, 3]

and diff. on (0, 3)

Solution (b) The average, then f'(c)



(b)

Plan: avg. → f'(c) = avg.

$f(3) = 27 - 6 = 21, \quad f(0) = 0$

Your turn: the average.

$$\frac{f(3) - f(0)}{3 - 0} = \frac{21 - 0}{3} = 7$$

$$f'(x) = 3x^2 - 2 = 7$$

$$3x^2 = 9 \rightarrow x^2 = 3 \rightarrow x = \pm\sqrt{3}$$

$-\sqrt{3} < 0$ → out; $0 < \sqrt{3} < \sqrt{9} = 3$ → in

$c = \sqrt{3}$

AP 2025 Q1(b)

Workbook: every c → keep inside

PROBLEM 2 OF 3

One table, three theorems

$W(t)$ is the water temperature, in °C, at time t minutes. W is differentiable. Justify each answer.

(a) Must $W'(t) = 4$ for some t in $(0, 3)$?

(b) Must $W'(t) = 0$ for some t in $(3, 12)$?

(c) Must $W(t) = 40$ for some t in $(0, 7)$?

t	0	3	7	12
W(t)	20	32	44	32

Solution (a) A value of W': the MVT

(a)

Plan: W' = avg. → MVT (Rolle if equal); W → IVT

Differentiable → cont. on [0, 3]

and diff. on (0, 3)

$$\frac{W(3) - W(0)}{3 - 0} = \frac{32 - 20}{3} = 4$$

MVT → at least one t in (0, 3)

with W'(t) = 4. Yes.

Your turn: part b.

Solution (b) Equal values: Rolle

t	0	3	7	12
W(t)	20	32	44	32

(b)

Plan: equal values → Rolle

[3, 12] → same reason

W(3), W(12) → 32

Rolle → at least one t in (3, 12)

W'(t) = 0. Yes. (MVT, too)

Your turn: part c, which theorem?

Solution (c) A value of W: the IVT

t	0	3	7	12
$w(t)$	20	32	44	32

(c)

Plan: a value of w → IVT

No prime: want w itself → the IVT

w cont. on [0, 7] (same reason)

20 < 40 < 44

IVT → at least one t in (0, 7)

with w(t) = 40. Yes.

Workbook: new table → theorem, reason

Which theorem?

✗ Common mistake

w cont. → IVT → $w' = 4$

No reason; wrong theorem

✓ Do this

want w' → MVT

IVT → a value of w

w diff. → cont.

AP 2023 Q1(b)

→ $f' = 0$ inside: MVT or Rolle

PROBLEM 3 OF 3

Which one meets the conditions?

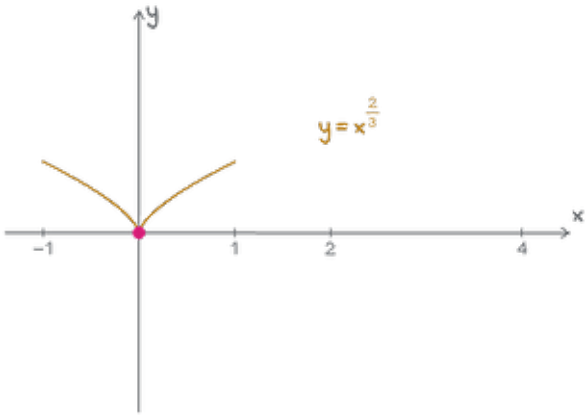
Which function satisfies the hypotheses of the Mean Value Theorem on the given interval?

(A) $f(x) = |x - 2|$ on $[0, 4]$

(B) $f(x) = \dfrac{1}{x - 1}$ on $[0, 2]$

(C) $f(x) = x^{\frac{2}{3}}$ on $[-1, 1]$

(D) $f(x) = \sqrt{x}$ on $[0, 4]$



Solution Both conditions, choice by choice

Plan: [a, b] cont., (a, b) diff. → each choice

(A) → corner at 2, inside → no

(B) → breaks at 1, inside → no

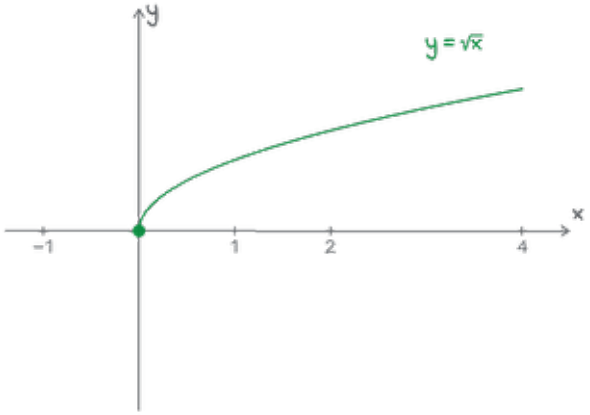
(C) $x^{\frac{2}{3}}$ → a sharp point at 0

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} = \frac{2}{3\sqrt[3]{x}}$

$f'(0)$ DNE, 0 inside $(-1, 1)$ → no. Three out.

Your turn: check (D).

Solution Both conditions for (D)



Plan: both conditions for (D)

(D) $\sqrt{x}$ → cont. on $[0, 4]$

$$f'(x) = \frac{1}{2\sqrt{x}}$$

No f' at 0 only → an endpoint

$(0, 4)$ leaves out 0 → differentiable

So (D).

Workbook: a new list → watch endpoints

WRAP-UP

Exactly 50? Yes

Check first: continuous on $[a, b]$, differentiable on (a, b)

At least one c: slope = average → yes, exactly 50

Reason first: differentiable, so continuous; fails → no promise

f itself → IVT (height); f' = average → MVT (slope)

Workbook first → 5.2 Extreme Value Theorem

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.