4.7 / L'Hospital's Rule

Watch the lesson ➤

Using L'Hospital's rule for determining limits of indeterminate forms

Learning goal

Use L'Hospital's rule for 0/0 and infinity/infinity, writing each limit first, and only when the form is indeterminate.

Key ideas

01 $\lim_{x\to 0}\dfrac{\sin 3x}{e^x-1}$: the 0/0 form, and factoring or a conjugate does **nothing**.

02 Near a, $f(x)\approx f'(a)(x-a)$ and $g(x)\approx g'(a)(x-a)$ (two tangent lines), so $\dfrac{f(x)}{g(x)}\approx\dfrac{f'(a)}{g'(a)}$. L'Hospital's rule: if $\lim f(x)=0$ and $\lim g(x)=0$ (or both $\pm\infty$), then $\lim_{x\to a}\dfrac{f(x)}{g(x)}=\lim_{x\to a}\dfrac{f'(x)}{g'(x)}$.

03 Write each limit alone: "0/0" is a label, not an equation. For ∞/∞: $\dfrac{f}{g}=\dfrac{1/g}{1/f}$ is the 0/0 form, so the same rule, just a **flip**.

04 $\lim_{x\to 0}\dfrac{\sin 3x}{e^x-1}=\lim_{x\to 0}\dfrac{3\cos 3x}{e^x}=3$. Where plugging in works, the rule gives a wrong answer (AP 2019). It is $\dfrac{f'}{g'}$, not $\left(\dfrac{f}{g}\right)'$: **no** quotient rule.

Remember

indeterminate form

0/0 or $\pm\infty/\pm\infty$: the limits of the top and the bottom alone do not decide the limit.

L'Hospital's rule

For an indeterminate form, $\lim\dfrac{f}{g}=\lim\dfrac{f'}{g'}$.

Class notes

4.7 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Not the quotient rule

Code R8

✗ $\lim \frac{f}{g} = \lim \left(\frac{f}{g}\right)' = \lim \frac{f'g - fg'}{g^2}$.

✓ Differentiate the top and the bottom separately: $\lim \frac{f'}{g'}$.

02 Write the two limits first

Code R9

✗ $\lim_{x \to 1} \frac{f(x) - 4}{x^2 - 1} = \frac{0}{0} = \lim \frac{f'(x)}{2x}$.

✓ $\lim(f(x) - 4) = 0$ and $\lim(x^2 - 1) = 0$, so the rule applies. Never write "$= \frac{0}{0}$".

4.7 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
0/0 with the limits written first	Write lim(top) = 0 and lim(bottom) = 0; then $\lim \frac{f'}{g'}$.	E01 · Practice 1, 2
The rule twice, or infinity over infinity	Check the form again after each use.	E02, E03 · Practice 3, 4
When the rule does not apply	Not 0/0 or $\pm\infty/\pm\infty$: plug in.	Practice 5

Example 01

Video · E01

f has a continuous derivative, with values in the table.

(a) Find $\lim_{x\to 1} \dfrac{f(x)-4}{x^2-1}$. Show that L'Hospital's rule applies.

(b) Find $\lim_{x\to 1} \dfrac{f(x)}{x+1}$.

x	1
$f(x)$	4
$f'(x)$	6

Plan: the top → 0, the bottom → 0, then the rule.

Example 02

Video · E02

$\lim_{x\to 0} \dfrac{e^{2x}-1-2x}{x^2} =$

A. 0
B. 1
C. 2
D. nonexistent

Check, use the rule, check again.

Example 03

Video · E03

Find $\lim_{x\to\infty} \dfrac{\ln(x^2+1)}{\ln x}$.

> Top and bottom both go to ∞; then the rule.

VIDEO COMPLETE Check your example answers at the end of this booklet.

4.7 / Example answers

L'Hospital's Rule

E01 (a) 3; (b) 2

E02 (C)

E03 2