4.6 / Practice workbook

Local Linearity and Linearization

4.6 / AP-style free response

Problem 1 ➤ */ Show the work and the reasons a grader looks for.*

A function f has $f(2) = 5$ and $f'(x) = x^2 - 1$.

(a) Write an equation of the line tangent to the graph of f at $x = 2$, and use it to approximate $f(2.1)$.

(b) Is the approximation an overestimate or an underestimate? Give a reason.

2 The tangent line to $y = \sqrt[3]{x}$ at $x = 8$ is used to approximate $\sqrt[3]{8.12}$. What is the approximation?

A. 2.01
B. 2.04
C. 2.12
D. 1.99

4.6 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

A function f is differentiable, with $f(1.5) = 3.2$ and $f'(1.5) = -2$. It is known that $f''(x) > 0$ for all x.

(a) Use the tangent line at $x = 1.5$ to approximate $f(1.6)$.

(b) Is your approximation greater than or less than $f(1.6)$? Give a reason.

4 The tangent line to $y = e^x$ at $x = 0$ is used to approximate $e^{0.1}$. Which of the following is true?

A. 1.1, an overestimate
B. 1.1, an underestimate
C. 1.01, an underestimate
D. 1.105, an overestimate

4.6 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

The local linear approximation of a differentiable function g at $x = 3$ is $L(x) = 4 - 2(x - 3)$.

(a) Find $g(3)$ and $g'(3)$.

(b) Approximate $g(3.2)$.

(c) Let $h(x) = x\,g(x)$. Use the tangent line to h at $x = 3$ to approximate $h(3.2)$.

4.6 / AP-style free response

Problem 6 ➤ */ Show the work and the reasons a grader looks for.*

The point $(2, 1)$ is on the curve $x^3 + 2y^2 = 10$. Use the line tangent to the curve at $(2, 1)$ to approximate the value of y on the curve near $(2, 1)$ when $x = 1.9$.

4.6 / Answer key

Local Linearity and Linearization

1 (a) $y = 5 + 3(x - 2)$; $f(2.1) \approx 5.3$; (b) underestimate

2 (A)

3 (a) 3; (b) less than

4 (B)

5 (a) $g(3) = 4$, $g'(3) = -2$; (b) $\frac{18}{5}$; (c) $h(3.2) \approx 11.6$

6 1.3