4.6 / Local Linearity and Linearization

Watch the lesson ➤

Approximating values of a function using local linearity

Learning goal

Approximate a value with the tangent line, and say whether it is an overestimate or an underestimate, with a reason from the picture or a given f".

Key ideas

01 $\sqrt{4.1}$ without a calculator? It is between 2 and 2.1; how close? Near 4, the curve is almost its tangent line.

02 $f(4) = 2$, $f'(x) = \frac{1}{2\sqrt{x}}$, $f'(4) = \frac{1}{4}$: $L(x) = 2 + \frac{1}{4}(x - 4)$, $L(4.1) = 2.025$ (square it: just over 4.1). The local linear approximation.

03 The curve is below the line, so L is too **high**: 2.025 is an overestimate. The slope shrinks, the curve falls below: over. $f'' > 0$: the slope grows, the curve is above: under.

04 The sign of f'' must hold on the **whole** interval from a to x.

Remember

local linear approximation
$f(x) \approx f(a) + f'(a)(x - a)$ for x near a.

Class notes

4.6 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Over or under needs a reason

Code R7

✗ $f(1.2) \approx 4.6$, an underestimate.

✓ Give the reason: $f''(x) > 0$ on $[1, 1.2]$, so the curve is above the tangent line.

4.6 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Approximate with the tangent line	$L(x) = f(a) + f'(a)(x - a)$; put in the x near a.	E03 · Practice 2, 5, 6
Over or under, with a reason	Curve below the line: over. Curve above: under. Given $f'' > 0$ on the interval: under; $f'' < 0$: over.	E01, E02 · Practice 1, 3, 4

Example 01

Video · E01

A function f has $f(1) = 4$ and $f'(x) = \sqrt{x^3 + 8}$.
(a) Use the tangent line at $x = 1$ to approximate $f(1.2)$.
(b) Is it an underestimate or an overestimate? Explain.

> L at $x = 1$, then the sign of f''.

Example 02

Video · E02

The tangent line to $y = \sqrt[3]{x}$ at $x = 27$ is used to approximate $\sqrt[3]{26.7}$. The approximation and its type are

A. $\frac{269}{90}$, an overestimate
B. $\frac{269}{90}$, an underestimate
C. $\frac{271}{90}$, an overestimate
D. $\frac{271}{90}$, an underestimate

> L at 27, then which way the curve bends.

Example 03

Video · E03

The point $(1, 2)$ is on the curve $x^3 + y^3 = 9$. Use the tangent line at $(1, 2)$ to approximate the y-value on the curve when $x = 1.1$.

> The slope at $(1, 2)$ first.

VIDEO COMPLETE Check your example answers at the end of this booklet.

4.6 / Example answers

Local Linearity and Linearization

E01 (a) $f(1.2) \approx 4.6$; (b) underestimate

E02 (A)

E03 $y \approx 1.975$