4.5b / Related Rates: Similar Triangles

Watch the lesson ➤

Solving related rates problems with cones and shadows · Part 2

Learning goal

Use similar triangles to write one quantity in terms of another before differentiating, for cones, troughs and shadows.

Key ideas

01 A cone, point down, filled at 2 ft³/min: does the level rise at a steady rate? Low down it is narrow, so a thick layer; high up it is wide, so a **thin** layer. The level slows down.

02 r and h both move. Similar triangles: $\frac{r}{h} = \frac{4}{10}$, so $r = \frac{2}{5}h$, true at every moment. Fewer letters, fewer headaches: $V = \frac{1}{3}\pi \cdot \frac{4h^2}{25} \cdot h = \frac{4\pi}{75}h^3$.

03 Chain rule: $\frac{d}{dt}h^3 = 3h^2\frac{dh}{dt}$, so $\frac{dV}{dt} = \frac{4\pi}{25}h^2\frac{dh}{dt}$.

04 Always true, before $\frac{d}{dt}$; one moment, after. If it moves, it is a letter (AP 2022: r held constant).

Remember

similar triangles
Triangles with the same angles: their sides keep the same ratio, at every moment.

Class notes

4.5b / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 A moving radius is not a number

Code R6

✗ At $h = 4$, $r = 2$, so $V = \frac{1}{3}\pi(2)^2 h$ and $\frac{dV}{dt} = \frac{4}{3}\pi\frac{dh}{dt}$.

✓ r changes with h: use $r = \frac{h}{2}$ before $\frac{d}{dt}$.

4.5b / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
A cone of water (similar triangles first)	Similar triangles: r in terms of h (true at every moment); then $V(h)$; then $\frac{d}{dt}$.	E01, E03 · Practice 1, 5
A shadow (similar triangles in a picture)	Two similar triangles; solve for the shadow; then $\frac{d}{dt}$. The tip is at $x + s$.	E02 · Practice 2
A trough or a pile (another shape, same idea)	Write the volume with one changing letter, then $\frac{d}{dt}$.	Practice 3, 4
Two rates given (keep both letters, product rule)	When the problem gives the rate of r and the rate of h, keep both letters and use the product rule on r^2h.	Practice 6

Example 01

Video · E01

A tank is a cone, point down, with radius 5 ft at the rim and height 10 ft. Water flows in at 2 ft³/min. How fast is the water level rising when the water is 4 ft deep?

> r in terms of h first, then $V(h)$, then $\frac{d}{dt}$.

Example 02

Video · E02

A 6 ft person walks away from a 15 ft streetlight at 5 ft/s. How fast is the tip of the shadow moving?

A. $\frac{10}{3}$ ft/s
B. 5 ft/s
C. $\frac{25}{3}$ ft/s
D. 2 ft/s

> The tip is at $x + s$.

Example 03

Video · E03

A block of ice is shaped like a cone. Right now its radius is 6 cm and its height is 10 cm. As it melts, the radius is decreasing at $\frac{1}{2}$ cm/h and the height is decreasing at 1 cm/h. $\left(V = \frac{1}{3}\pi r^2 h\right)$
(a) How fast is the volume changing right now?
(b) Is the volume increasing or decreasing?

> Plan: two rates → keep r, h → product rule

VIDEO COMPLETE Check your example answers at the end of this booklet.

4.5b / Example answers

Related Rates: Similar Triangles

E01 $\frac{1}{2\pi} \approx 0.16$ ft/min

E02 (C)

E03 (a) $\frac{dV}{dt} = -32\pi$ cm^3/hr; (b) decreasing, at 32π cm^3/hr