4.5a / Related Rates: Right Triangles and Angles

Watch the lesson ➤

Solving related rates problems with right triangles · Part 1

Learning goal

Set up right-triangle and angle related rates, with the right signs for toward and away, and differentiate before putting in numbers.

Key ideas

01 A ladder: the foot slides out at 2 ft/s. Does the top slide down at 2 ft/s? $x = 7 \to 15$ gives $y = 24 \to 20$: out 8, down only 4. $x = 20 \to 24$ gives $y = 15 \to 7$: out 4, down 8. Not the **same**.

02 $x^2 + y^2 = 25^2$; 25 is a constant, so no $\frac{d}{dt}$ on it: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, $\frac{dy}{dt} = -\frac{x}{y} \cdot \frac{dx}{dt}$. As x grows and y shrinks, $\frac{x}{y}$ grows: the top **speeds** up.

03 Signs: toward is negative, away is positive (AP 2022 lost points writing +2 for a decrease).

04 An angle: a kite at 30 m, x along the ground: $\tan\theta = \frac{30}{x}$ (opposite over adjacent). $\frac{d}{dt}\tan\theta = \sec^2\theta \cdot \frac{d\theta}{dt}$, so $\sec^2\theta\,\frac{d\theta}{dt} = -\frac{30}{x^2} \cdot \frac{dx}{dt}$.

Remember

rate toward
A distance that shrinks: its rate is negative.

Class notes

4.5a / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 A decreasing distance has a negative rate

Code R5

✗ Car A drives toward the intersection at 30 mph, so $\frac{dy}{dt} = 30$.

✓ y is shrinking: $\frac{dy}{dt} = -30$.

4.5a / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
The Pythagorean theorem in time	Name the legs; $x^2 + y^2 = s^2$; $\frac{d}{dt}$; toward is negative, away is positive.	E01, E02 · Practice 1, 2, 4, 5, 6
An angle that changes	A trig ratio with the fixed side; $\frac{d}{dt}$ with the chain rule on θ.	E03 · Practice 3

Example 01

Video · E01

A boat is pulled toward a dock by a rope. The rope runs from the boat to a ring on the dock, 5 ft above the boat. The rope is pulled in at 2 ft/s. How fast is the boat approaching the dock when 13 ft of rope is out?

> The 5 ft side is fixed: $x^2 + 5^2 = s^2$. The rope s shrinks: $\frac{ds}{dt} = -2$.

Example 02

Video · E02

Car A is 6 mi north of an intersection, driving toward it at 30 mph. Car B is 8 mi east of it, driving away at 40 mph.
(a) How fast is the distance between the cars changing?
(b) Is the distance increasing or decreasing?

> Toward is negative; away is positive.

Example 03

Video · E03

A balloon rises straight up at 5 m/s from a point 100 m from an observer. How fast is the angle of elevation θ increasing when the balloon is 100 m high?

$\tan\theta = \frac{h}{100}$, then $\frac{d}{dt}$.

A. $\frac{1}{20}$ rad/s

B. $\frac{1}{40}$ rad/s

C. $\frac{1}{10}$ rad/s

D. $\frac{1}{100}$ rad/s

VIDEO COMPLETE Check your example answers at the end of this booklet.

4.5a / Example answers

Related Rates: Right Triangles and Angles

E01 $\frac{dx}{dt} = -\frac{13}{6}$: the boat approaches the dock at $\frac{13}{6}$ ft/s

E02 (a) $\frac{ds}{dt} = 14$ mph; (b) increasing

E03 (B)