4.4 / Practice workbook

Introduction to Related Rates

Practice

Problems 1–6 / Show your reasoning

1 A point moves along the curve $y = \sqrt{x + 7}$. When $x = 2$, $\frac{dx}{dt} = 6$ units per second. Find $\frac{dy}{dt}$ at that moment.

2 A stone dropped in a pond makes a circular ripple whose radius increases at 3 cm/s. How fast is the area inside the ripple increasing when the radius is 10 cm?

A. 6π cm^2/s
B. 30π cm^2/s
C. 60π cm^2/s
D. 300π cm^2/s

4.4 / AP-style free response

Problem 3 ➤ */ Show the work and the reasons a grader looks for.*

The radius of a cylinder is increasing at 0.5 cm/s while its height is decreasing at 2 cm/s. ($V = \pi r^2 h$)

(a) Find $\frac{dV}{dt}$ when $r = 3$ cm and $h = 10$ cm.

(b) Is the volume increasing or decreasing at that moment? Give a reason.

4 A particle moves along the curve $x^2y = 12$. At the point $(2, 3)$, $\frac{dx}{dt} = 3$. What is $\frac{dy}{dt}$ at that moment?

A. -9
B. 9
C. -3
D. -36

4.4 / AP-style free response

Problem 5 ➤ */ Show the work and the reasons a grader looks for.*

In a circuit, $V = IR$, where the voltage $V = 12$ volts stays constant. The resistance R is increasing at 0.5 ohm per second. How fast is the current I changing when $R = 4$ ohms?

4.4 / AP-style free response

Problem 6 ➤ / *Show the work and the reasons a grader looks for.*

Two resistors, with resistances x ohms and y ohms, are connected in parallel. The total resistance is $R = \dfrac{xy}{x+y}$ ohms. The resistance x is increasing at 1 ohm per second and y is increasing at 2 ohms per second.

(a) How fast is R changing when $x = 4$ and $y = 12$? Indicate units of measure.

(b) A student substitutes $x = 4$ and $y = 12$ into R first and gets $R = 3$, so $\dfrac{dR}{dt} = 0$. Explain the mistake.

4.4 / Answer key

Introduction to Related Rates

1 1

2 (C)

3 (a) 12π cm^3/s; (b) increasing

4 (A)

5 $\frac{dI}{dt} = -\frac{3}{8}$ ampere per second: decreasing

6 (a) $\frac{dR}{dt} = \frac{11}{16}$ ohm per second; (b) $R = 3$ is the value at one instant, a constant number, so its derivative is 0; R itself is changing. Differentiate first, then put in the numbers.