Lecture Notes · Unit 4 · Lecture 5 AP CED 4.4

Related rates

Tap a blue play button next to a heading to open that part of the video. ▶ Watch the video

INTRO: A GROWING CIRCLE

Related Rates AP Calculus 4.4

Radius → 2 cm/s

Area → 2 cm² every second, too?

Test it → and when to plug in numbers

Let's get that 5

IDEAS 1–2: BOTH CHANGE, DIFFERENTIATE WITH RESPECT TO T

Key ideas

Radius grows → $\frac{dr}{dt} = 2$ cm/s

So $\frac{dA}{dt} = 2$ cm²/s? Same number.

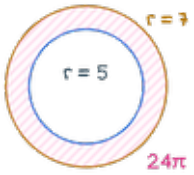
1. Both change with time

$A = \pi r^2$ (r changes)

r = 5 → A = 25π

1 s later, r = 7 → A = 49π

Change → 24π, not 2



2. Differentiate with respect to time

$dA = d(\pi r^2)$

$A = \pi r^2 : t$?

r depends on t → chain rule

$dA = 2\pi r\, dr$

$\div dt \rightarrow \frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}$

r = 5 → 2π(5)(2) = 20π cm²/s

One equation → related rates

IDEAS 3–4: EVERY VARIABLE GETS ITS D, NUMBERS LAST

Key ideas

3. Every changing variable gets its d

$y^2 \rightarrow 2y\,dy \rightarrow 2y\frac{dy}{dt}$

$y = f(x)$: $1\,dy = f'(x)\,dx \rightarrow \frac{dy}{dt} = f'(x)\frac{dx}{dt}$

$\frac{d}{dt}$ doesn't play favorites

Usually t → any variable they all share

4. Equation → d → numbers

Equation → d → numbers

Plug in r = 5 early → A = 25π

→ $\frac{dA}{dt} = 0$ → frozen

Stays fixed → a number, d = 0 · changes → waits

PROBLEM 1 OF 3

A particle on a curve

A particle moves along the curve $xy^2 - x = 6$. At the point $(2, 2)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that moment.

Product rule
$(uv)' = u'v + uv'$
Every variable gets its d
$y^2 \to 2y\,dy$

Solution Every variable gets its d, divide by dt

Plan: d each variable → ÷dt → numbers

$xy^2 \to u = x,\ v = y^2$

$$dx \cdot y^2 + x \cdot 2y\,dy - dx = 0$$

$$\frac{dx}{dt}y^2 + x \cdot 2y\frac{dy}{dt} - \frac{dx}{dt} = 0$$

Your turn: plug in.

Solution Numbers last

From the last screen

$$\frac{dx}{dt}y^2 + x \cdot 2y\frac{dy}{dt} - \frac{dx}{dt} = 0$$

Plan

$x = 2,\quad y = 2,\quad \frac{dx}{dt} = 3 \to$ solve

$$3(4) + 2(2)(2)\frac{dy}{dt} - 3 = 0$$

$$12 + 8\frac{dy}{dt} - 3 = 0 \to 8\frac{dy}{dt} = -9$$

So $\frac{dy}{dt} = -\frac{9}{8} \to y$ decreasing

Workbook → every term

Careful → product rule

X Common mistake

$$\frac{dx}{dt} \cdot 2y\frac{dy}{dt}$$

Derivatives multiplied

✓ Do this

$(uv)' = u'v + uv'$

$$\frac{dx}{dt}y^2 + 2xy\frac{dy}{dt}$$

PROBLEM 2 OF 3

A height from a table

The height of a tree is $H = f(x)$ feet, where x is the diameter of its trunk, in inches. The table gives values of f and f'. The diameter is growing at $\frac{1}{2}$ inch per year. How fast is the height changing when $x = 10$? Include units.

x (in)	8	10	12
f (ft)	30	39	46
f′ (ft/in)	5	4	3

Solution

H = f(x): differentiate each variable, divide by dt, then the table

Plan

$H = f(x) \to dH \to \div dt \to$ numbers

$$dH = f'(x)\,dx \to \frac{dH}{dt} = f'(x)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{1}{2}$$

$$\frac{dH}{dt} = 4 \cdot \frac{1}{2} = 2$$

Units → $\frac{ft}{in} \cdot \frac{in}{yr} = \frac{ft}{yr}$

At $x = 10$, height increasing 2 ft/yr

$y = f(x) \rightarrow$ workbook: keep $\frac{dx}{dt}$

PROBLEM 3 OF 3

A changing rectangle

A rectangle's length is increasing at 3 cm/s and its width is decreasing at 2 cm/s. When the length is 10 cm and the width is 6 cm,
(a) find the rate of change of the area.
(b) Is the area increasing or decreasing?

Product rule, one variable at a time
$d(LW) = dL \cdot W + L \cdot dW$

Solution A = LW: d of each variable, divide by dt

Plan: $A = LW \rightarrow$ d each variable $\rightarrow \div dt$

(a)

Shrinking $\rightarrow \frac{dW}{dt} = -2, \frac{dL}{dt} = 3$

$$dA = dL \cdot W + L \cdot dW$$

$$\frac{dA}{dt} = \frac{dL}{dt}W + L\frac{dW}{dt}$$

$$= 3(6) + 10(-2) = 18 - 20 = -2 \text{ cm}^2/\text{s}$$

Your turn: part (b) → area increasing or decreasing?

Solution The sign of the rate

Product rule, one variable at a time
$d(LW) = dL \cdot W + L \cdot dW$

(b)

Plan: the sign of the rate

$\frac{dA}{dt} < 0 \rightarrow$ decreasing at 2 cm²/s

~~−2~~ with "decreasing"

workbook → shrinking rate < 0

WRAP-UP

2 cm²/s? No

$\frac{dA}{dt} = 2\pi r\frac{dr}{dt} \rightarrow$ not 2

$r = 5 \rightarrow 20\pi$ cm²/s

Equation → d → numbers

workbook → video → comments → next: 4.5 triangles

Next: the workbook for this topic (link in the video description). Stuck? Ask in the comments.