4.4 / Introduction to Related Rates

Watch the lesson ➤

Rates that are related through one equation

Learning goal

Differentiate one equation with respect to time, with the chain rule on every changing variable, and only then put in the numbers.

Key ideas

01 A circle with $\frac{dr}{dt} = 2$ cm/s: is $\frac{dA}{dt} = 2$ cm^2/s? At $r = 5$, $A = 25\pi$; a second later $r = 7$, $A = 49\pi$: a change of $24\pi \approx 75$, not **2**.

02 Chain rule: r depends on t: $dA = 2\pi r\, dr$, divide by dt: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$. At $r = 5$: $2\pi(5)(2) = 20\pi$ cm^2/s. One equation, **related** rates.

03 Every change gets a $\frac{d}{dt}$: $\frac{d}{dt}\left(y^2\right) = 2y\frac{dy}{dt}$; for $y = f(x)$, $1\, dy = f'(x)\, dx$, so $\frac{dy}{dt} = f'(x)\frac{dx}{dt}$.

04 Plug in $r = 5$ too early and $A = 25\pi$ is a constant: $\frac{dA}{dt} = 0$, frozen. Equation, $\frac{d}{dt}$, then the numbers; a fixed length has $\frac{d}{dt} = 0$ (AP 2018: no chain rule, at most 1 of 3 points).

Remember

related rates
Rates of quantities tied by one equation, found by differentiating it with respect to t.

Class notes

4.4 / Write it clearly

What earns the point on the AP Exam, and the mistakes that lose it.

Common mistakes

The mistake code is used in the answers and in My Mistakes

01 Numbers in before d/dt

Code R4

✗ At $r = 5$, $A = 25\pi$, so $\frac{dA}{dt} = 0$.

✓ Differentiate $A = \pi r^2$ first: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$; then put in $r = 5$.

4.4 / Guided examples

Watch, pause, and write the reasoning. Full solutions are in the video.

Problem types

Read the problem, name the type, then use its method

Type	Method	Problems
Differentiate one equation with respect to t (chain rule on every variable)	Write the equation; take $\frac{d}{dt}$ of every term; then put in the numbers.	E02 · Practice 1, 2
A product or a quotient in the equation	Use the product or quotient rule inside $\frac{d}{dt}$.	E01, E03 · Practice 3, 4, 5, 6

Example 01

Video · E01

A particle moves along the curve $xy^2 - x = 6$. At the point $(2, 2)$, $\frac{dx}{dt} = 3$. Find $\frac{dy}{dt}$ at that moment.

> $\frac{d}{dt}$ every term, then plug in.

Example 02

Video · E02

The height of a tree is $H = f(x)$ feet, where x is the diameter of its trunk, in inches. The table gives values of f and f'. The diameter is growing at $\frac{1}{2}$ inch per year. How fast is the height changing when $x = 10$? Include units.

x (in)	8	10	12
$f(x)$ (ft)	30	39	46
$f'(x)$ (ft/in)	5	4	3

> Plan: $H = f(x) \to dH \to$ divide by dt $\to$ numbers

Example 03

Video · E03

A rectangle's length is increasing at 3 cm/s and its width is decreasing at 2 cm/s. When the length is 10 cm and the width is 6 cm,
(a) find the rate of change of the area.
(b) Is the area increasing or decreasing?

> $A = LW$, so the product rule in t.

VIDEO COMPLETE Check your example answers at the end of this booklet.

4.4 / Example answers

Introduction to Related Rates

E01 $-\frac{9}{8}$

E02 2

E03 (a) $\frac{dA}{dt} = -2$ cm^2/s; (b) decreasing